



# Massively Parallel Vortex Particle Simulations of Aircraft Wakes

Philippe Chatelain

with : M. Bergdorf, D. Rossinelli, P. Koumoutsakos  
and A. Curioni (IBM Research)

- Motivation
- Vortex Particle Method
- Medium Wavelength Instability
- Optimization of Wake Decay
- Conclusions

# Motivation

- **COMPUTATION**
  - Massively Parallel Simulations Using Particles
    - IBM/BG: Distributed memory ( $10^4 - 10^5$  CPUS), with low RAM per node
    - Achieve sustained  $O(\text{Tflops})$  performance

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- **PHYSICS**

- Optimization of aircraft wake decay

- aircraft & lift devices design

- High Reynolds DNS

- Capture accurate decay of vortical flows
- Physical insight - Basis for turbulent models

# Vortex Particle Method

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- **GOVERNING EQUATIONS**
  - Navier-Stokes, incompressible

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- **EVOLUTION EQUATIONS**

$$\begin{aligned}\frac{d\mathbf{x}_p}{dt} &= \mathbf{u}(\mathbf{x}_p) \\ \frac{d\alpha_p}{dt} &= ((\omega \cdot \nabla) \mathbf{u}(\mathbf{x}_p) + \nu \nabla^2 \omega(\mathbf{x}_p)) V_p\end{aligned}$$

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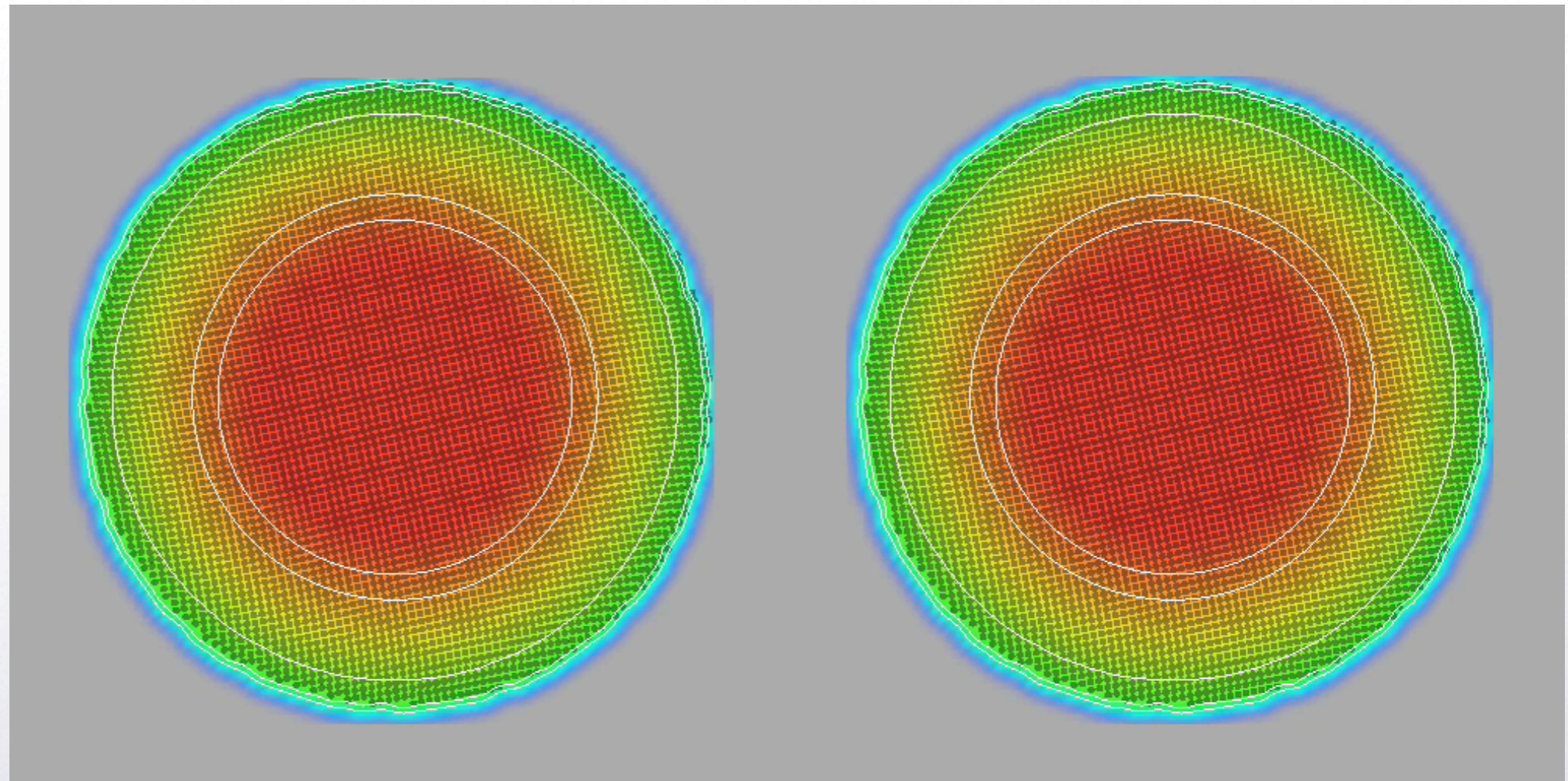
- Particle methods **ARE NOT MESH-FREE**

# Vortex Particle Method

- ACCURATE Particle methods ARE NOT MESH-FREE
  - ➔ Particle distortion = loss of convergence

$$\frac{D\Gamma_p}{Dt} = 0$$

Euler Equations (2D : u-w)  
for an incompressible evolution  
of an axi-symmetric vortex  
patch



# Remeshed Particle Methods

$$Q_p^{\text{new}} = \sum_{p'} Q_{p'} M(j h - x_{p'})$$

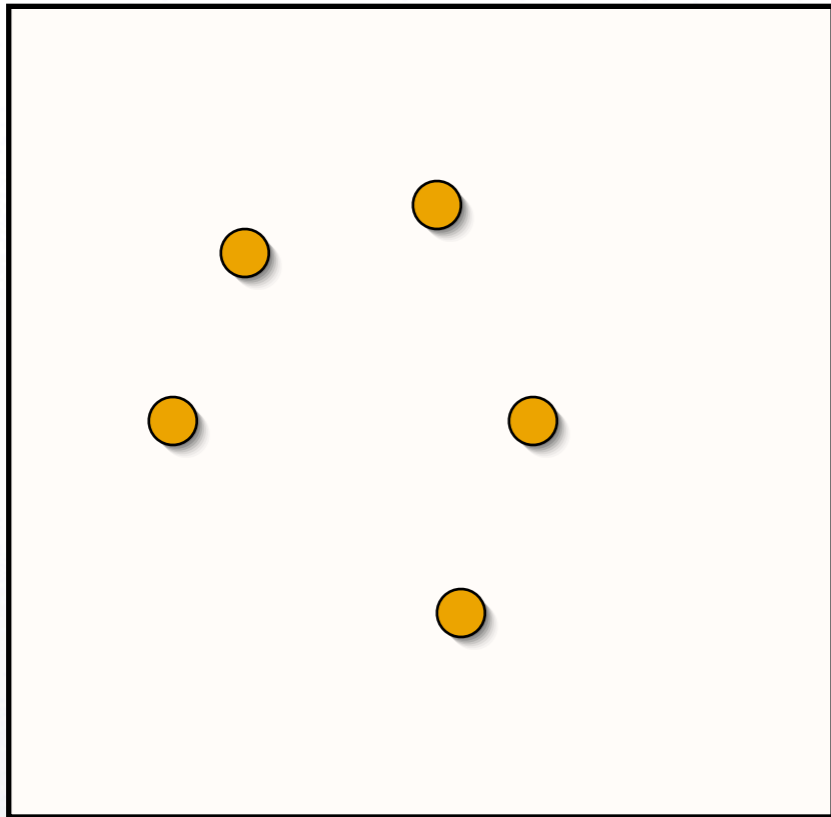
**Interpolation Kernel**  $M(x)$

- **Moment** conserving
- Tensorial Product of 1D kernels



# Remeshed Particle Methods

- *Remesh* : reinitialize particles onto regular locations



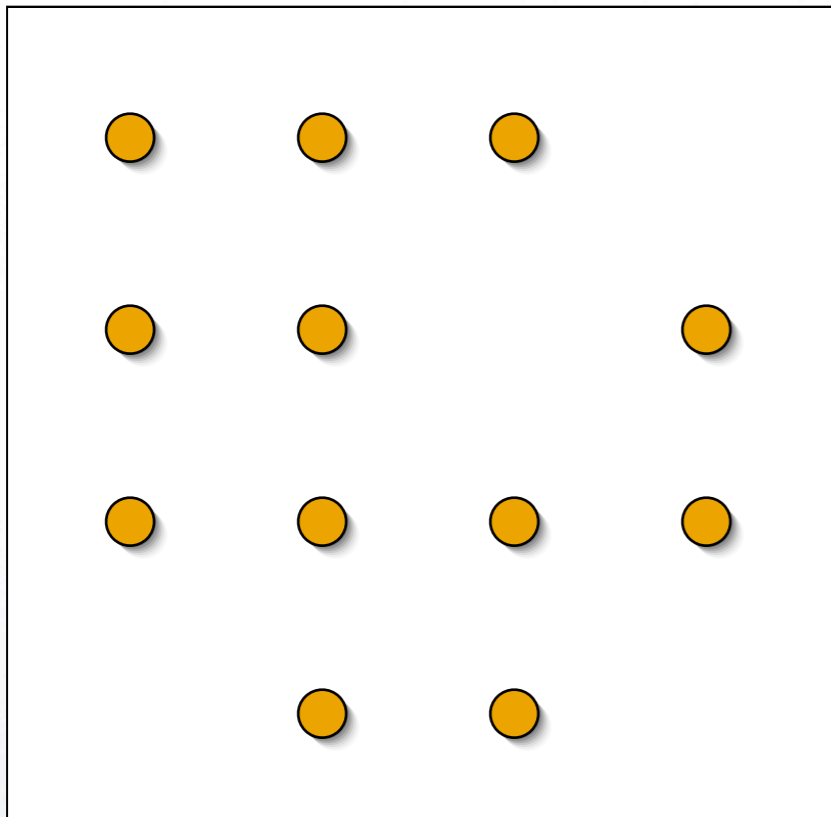
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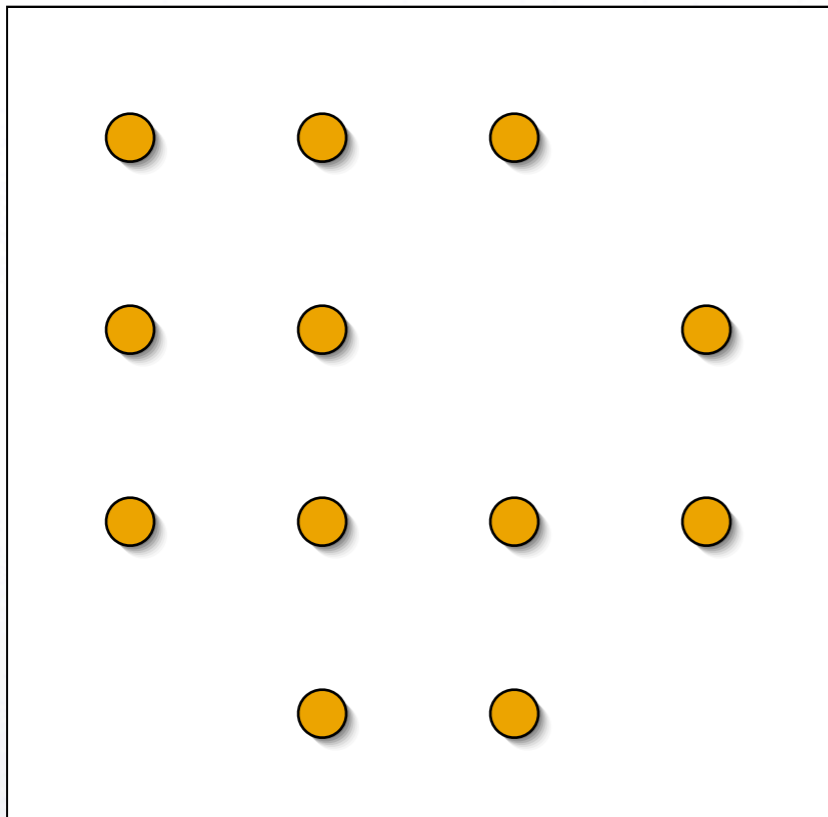
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- Mesh also used for the efficient computation of **Right-Hand Side**

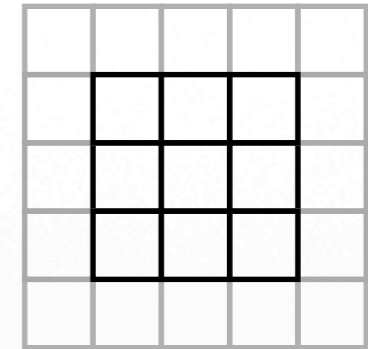
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$\omega_{ij}$

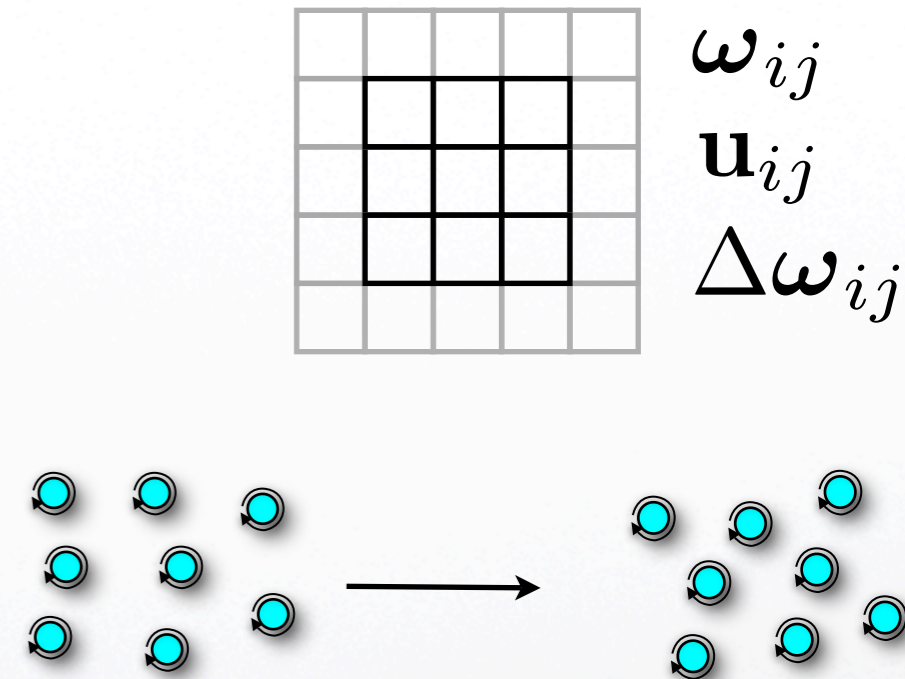
$\mathbf{u}_{ij}$

$\Delta\omega_{ij}$

# Vortex particle method

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- Particles only handle **advection**

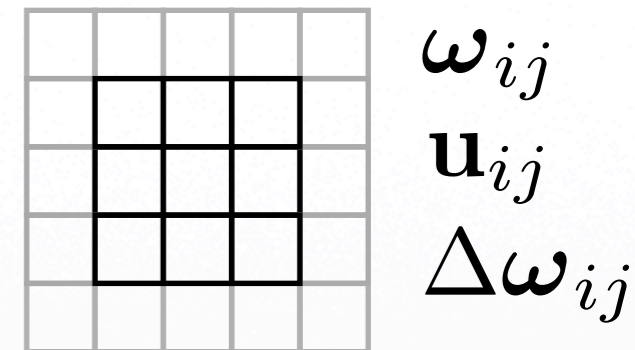
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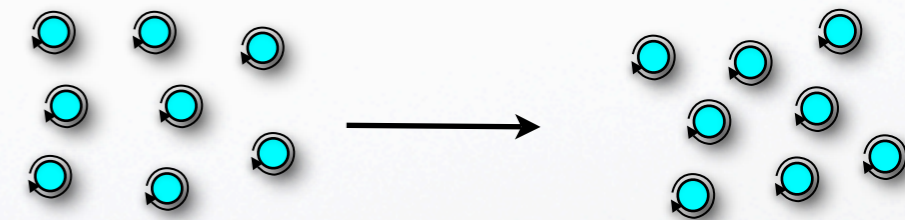
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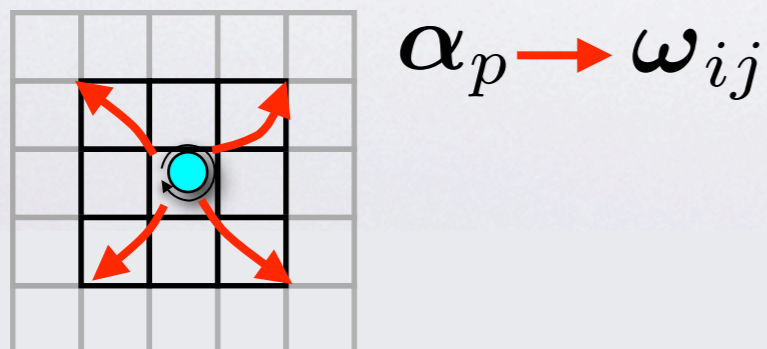


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- Particles and Mesh communicate through **interpolation**

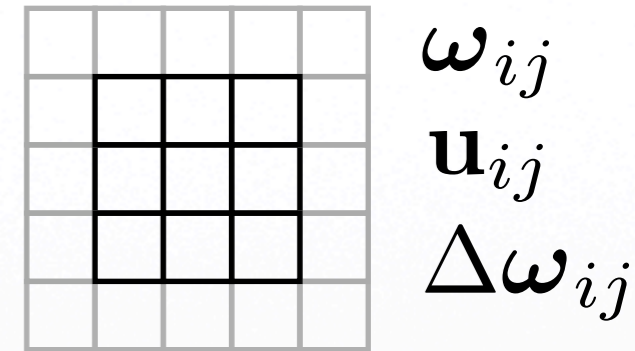




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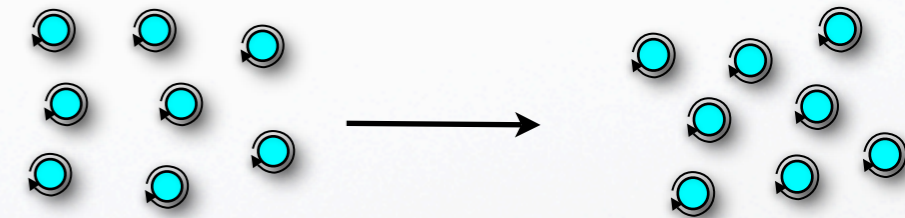
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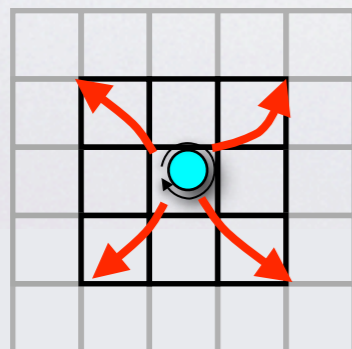


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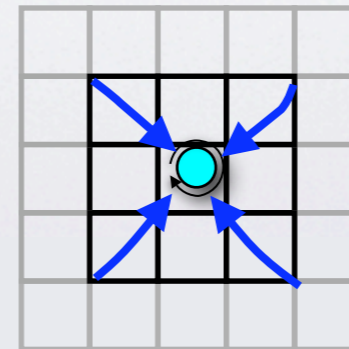
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- Particles and Mesh communicate through **interpolation**



$$\alpha_p \rightarrow \omega_{ij}$$



$$\begin{matrix} \mathbf{u}_{ij} \\ (\omega \cdot \nabla) \mathbf{u}_{ij} \\ \Delta\omega_{ij} \end{matrix} \rightarrow \begin{matrix} \mathbf{u}_p \\ (\omega \cdot \nabla) \mathbf{u}_p \\ \Delta\omega_p \end{matrix}$$

# Implementation

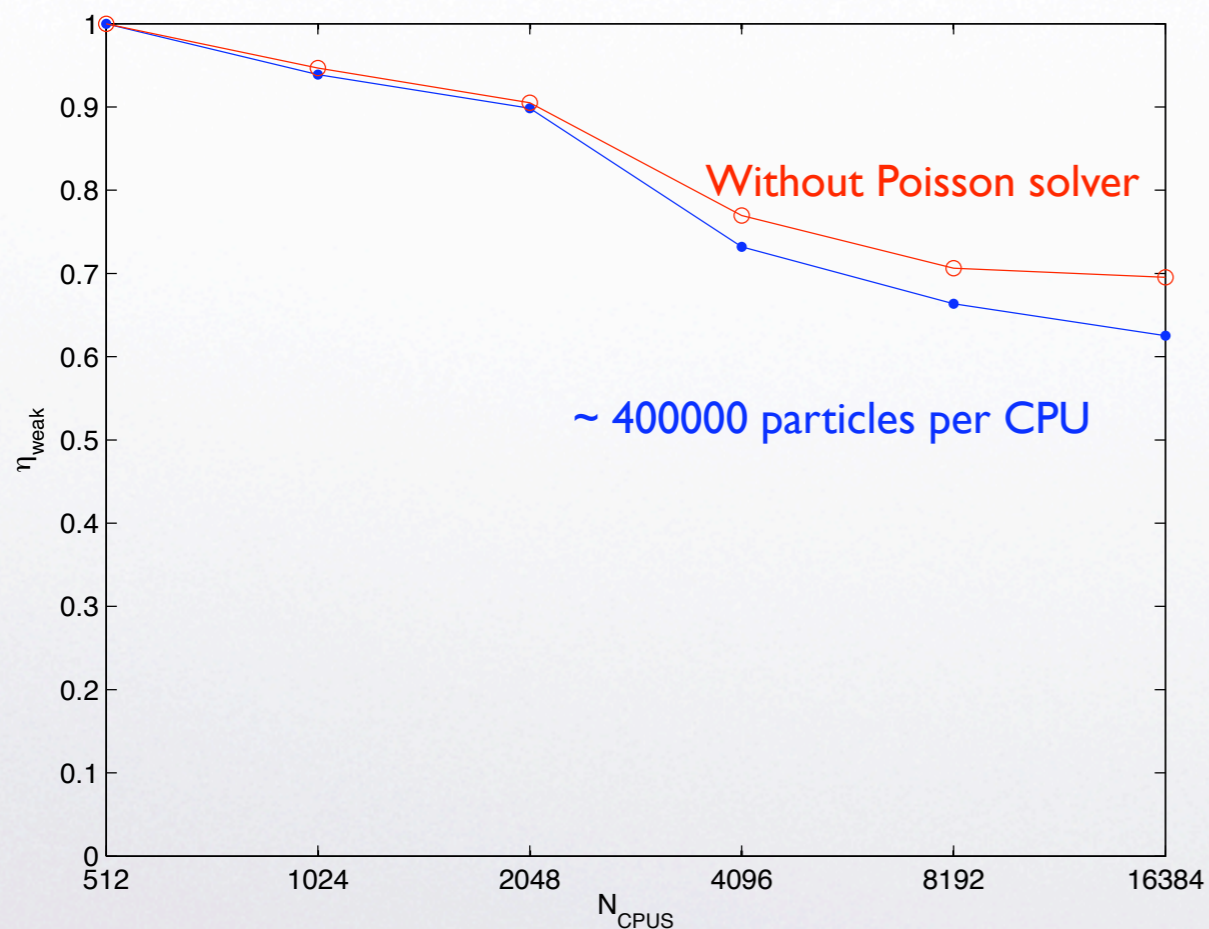
[www.cse-lab.ethz.ch/software.html](http://www.cse-lab.ethz.ch/software.html)

- Code is client of the **PPM** library
- **Parallel Particle Mesh Library** (Fortran 90, MPI)
- Client and library *tuned* for BG/L

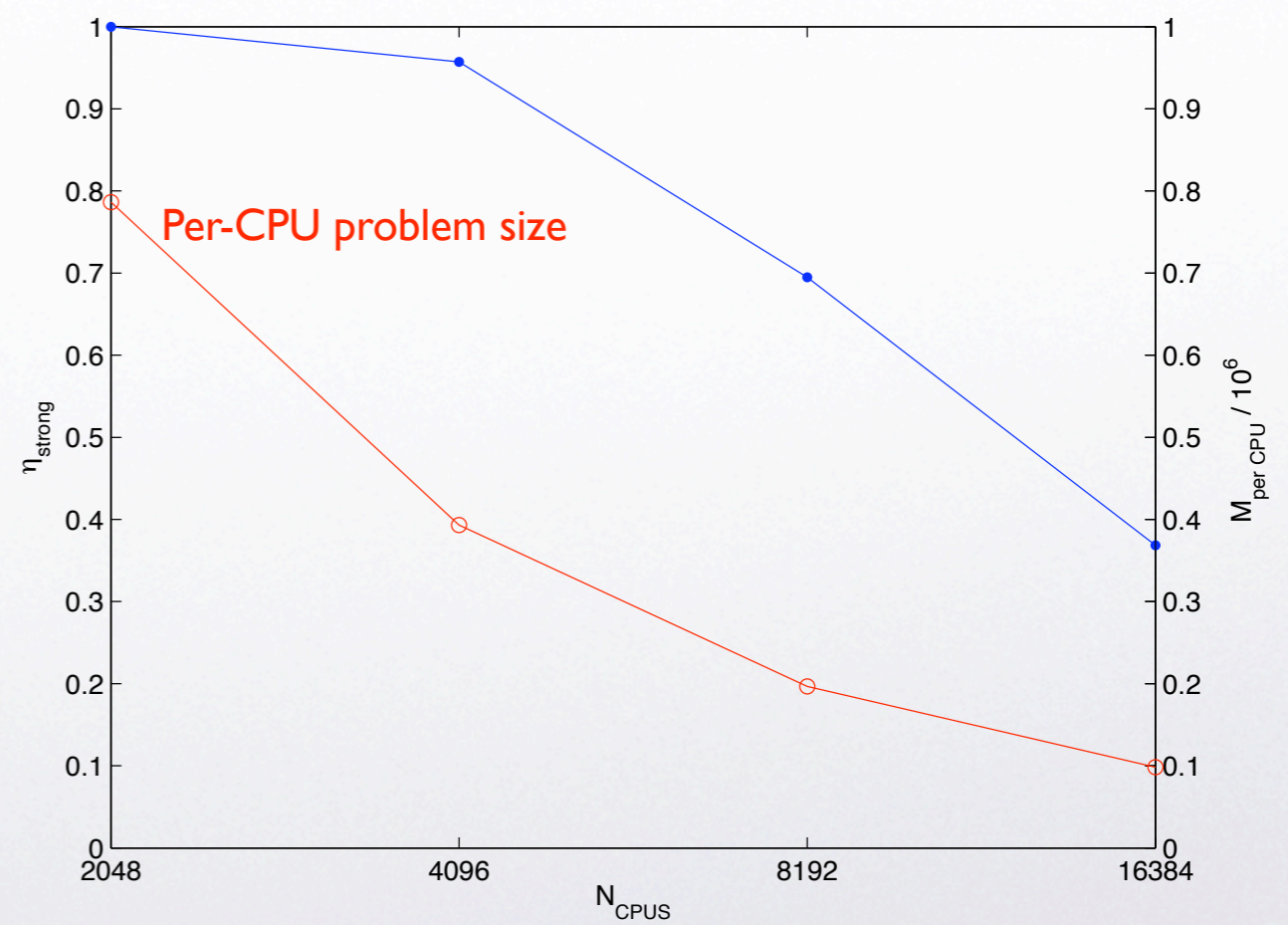
Sbalzarini et al, JCP 2006

Chatelain et al, CMAME 2008

### Weak eff.: constant size per CPU



### Strong eff.: constant total size



PPMers: I. Sbalzarini, J. Walther, M. Bergdorf, P. Chatelain, S. Hieber, E. Kotsalis, P. Koumoutsakos, F. Milde, M. Quack, B. Hejazi Alhosseini

IBM T. J. Watson Center, Yorktown Heights, NJ  
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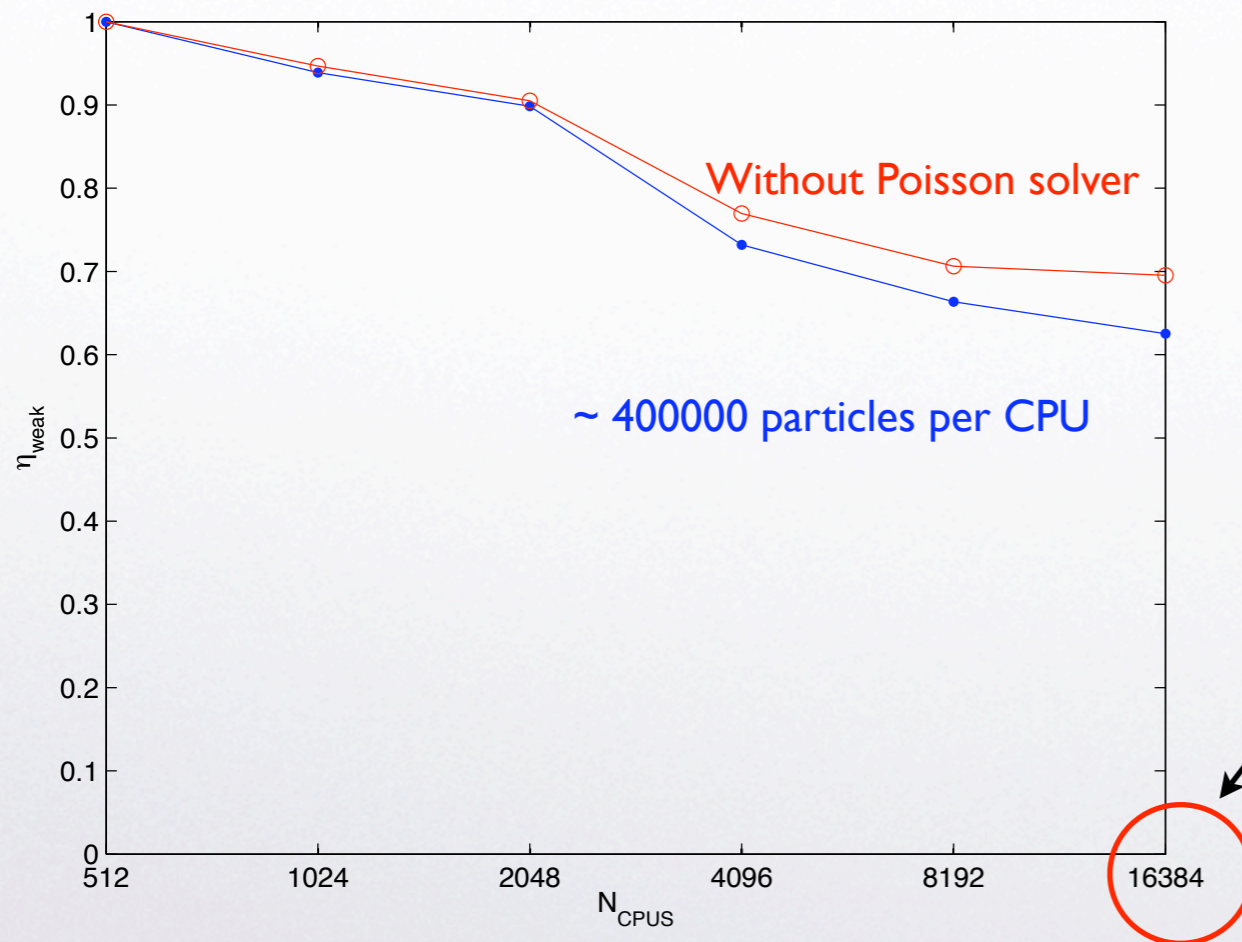
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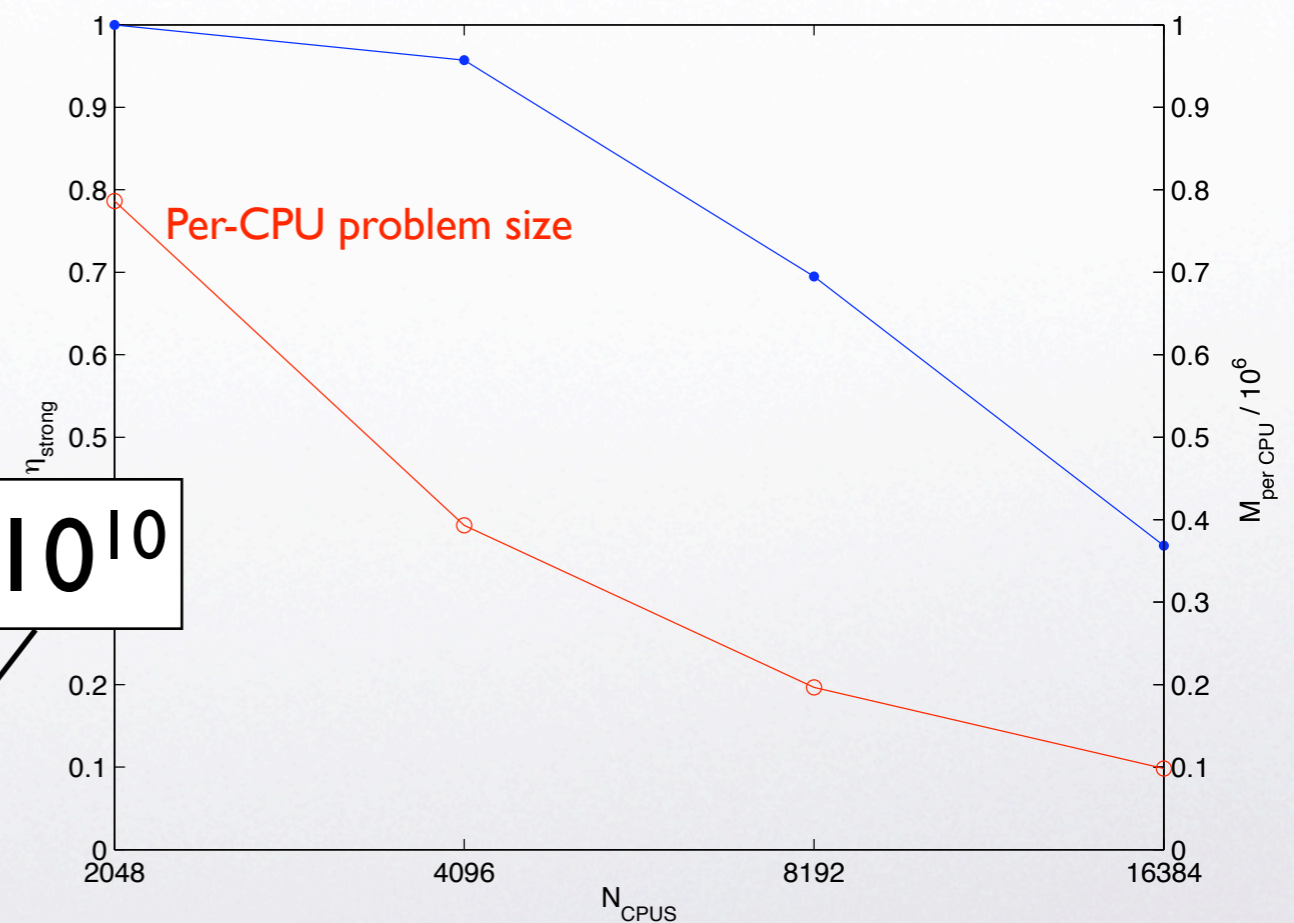
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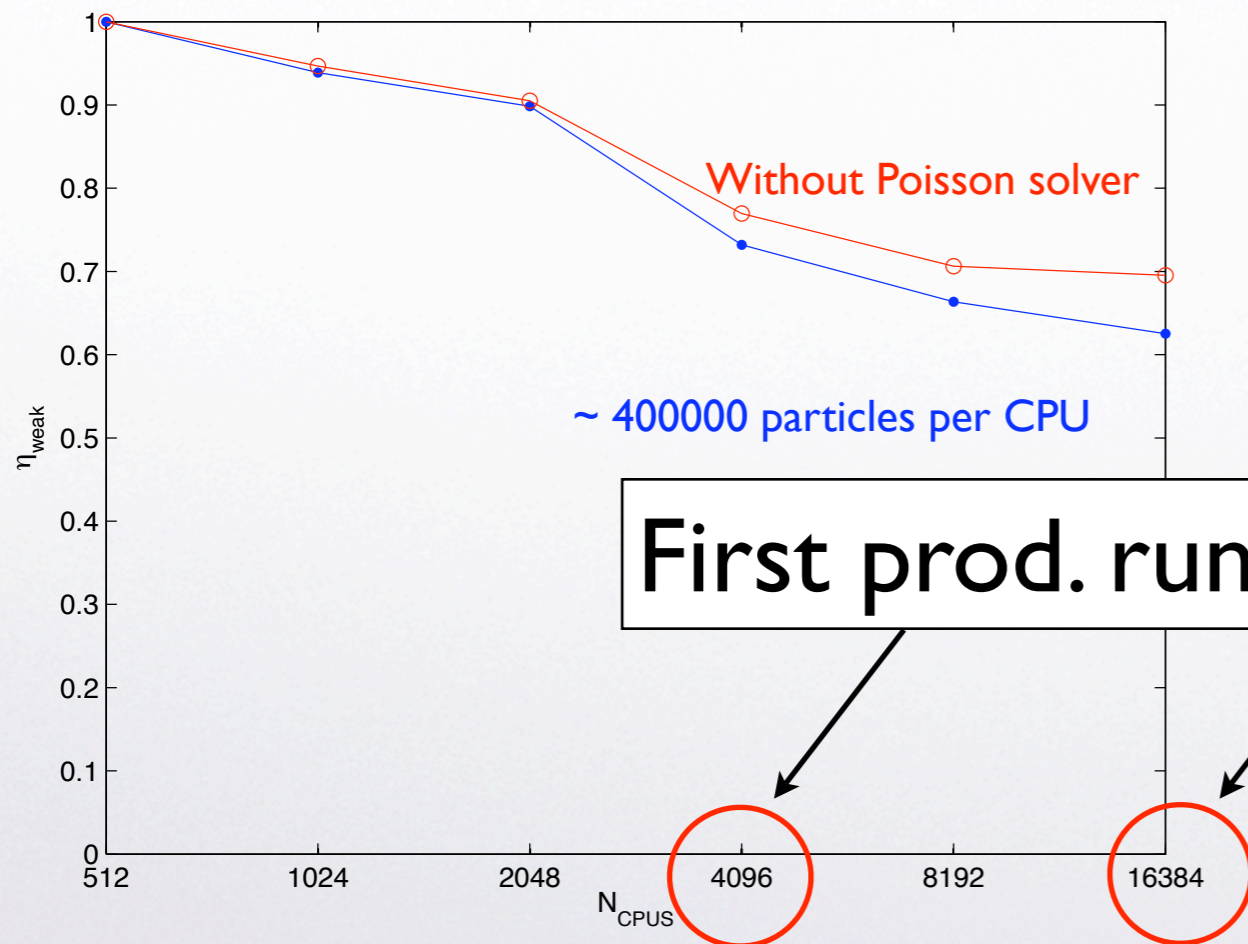
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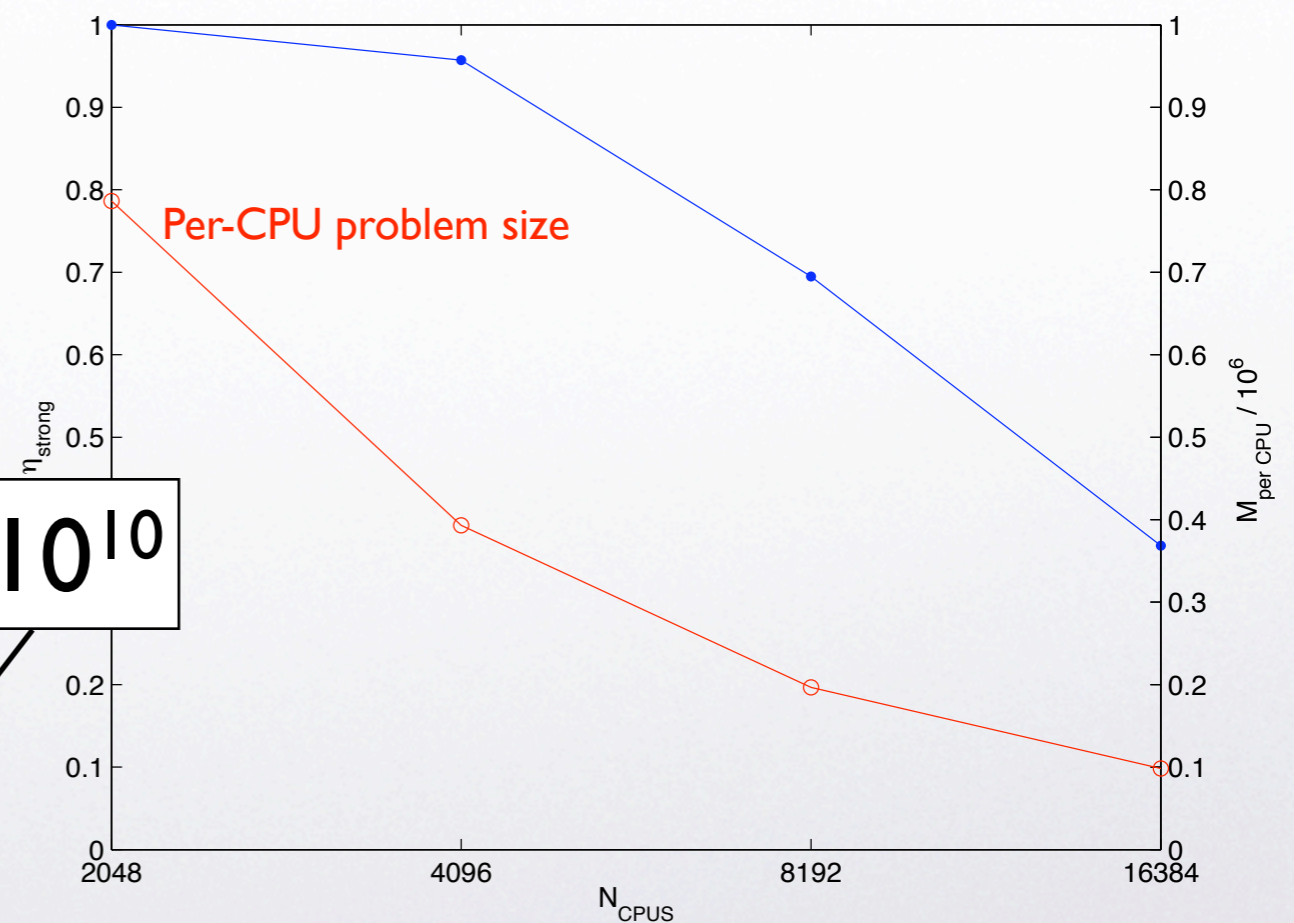
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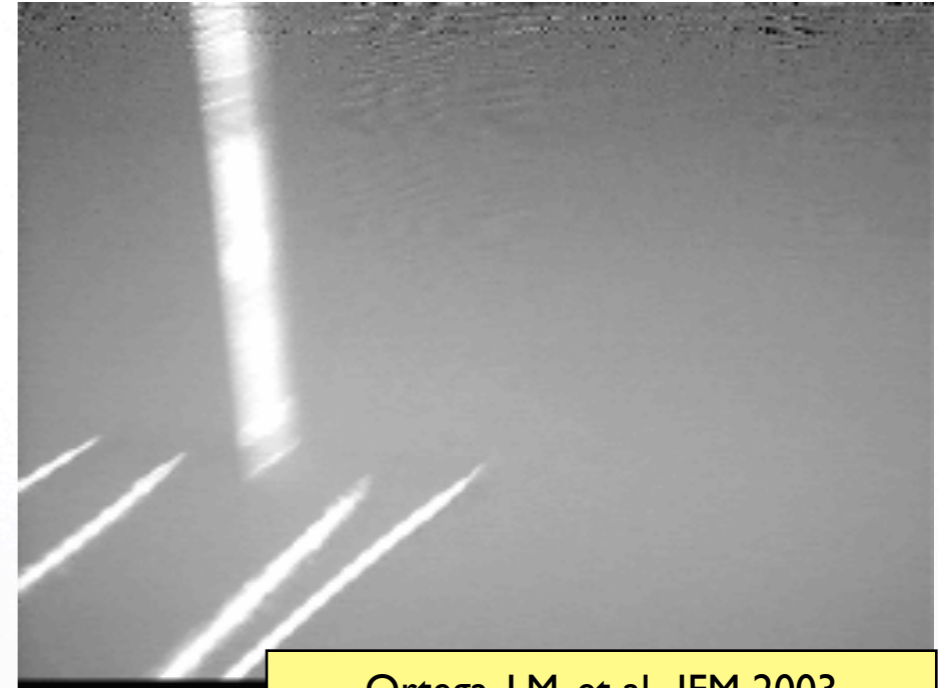
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- Long domain
- Periodic in all directions
- Initiation by ambient noise

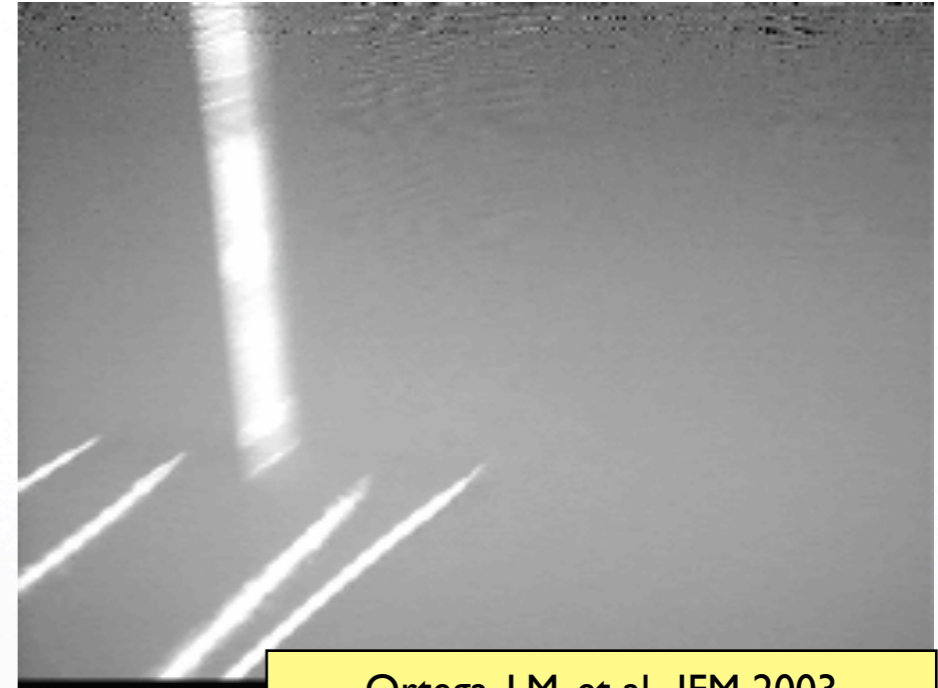


Ortega, J.M. et al., JFM 2003

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## Physical parameters

- $Re_{\Gamma} = 6,000$
- Circulation ratio  $\Gamma_2/\Gamma_1 = -0.35$
- Span ratio  $b_2/b_1 = 0.5$
- Domain  $L_x = 10 b_1$
- Time = 0.35
- Ambient white noise  $u_{RMS} = 0.5\%$

## Numerical parameters

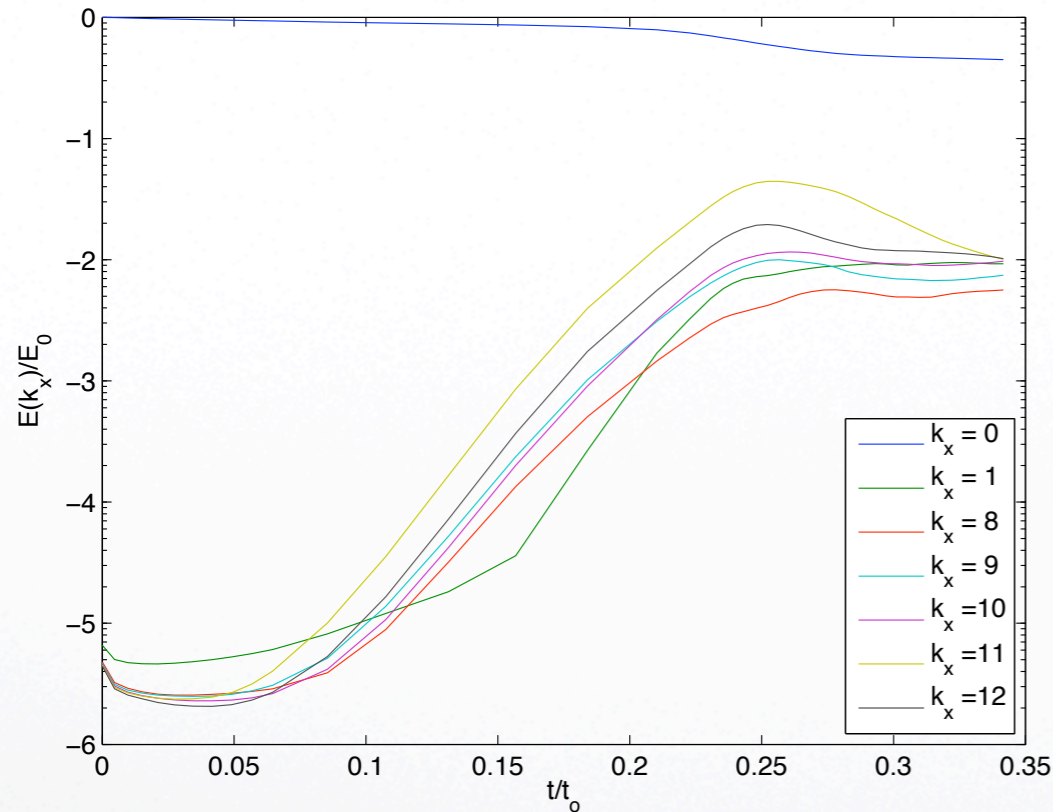
- $\sim 1.6 \cdot 10^9$  particles
- $1024 \times 768 \times 2,048$  grid
- 10,000 time steps
- RK3 Low-storage
- 4th order FD

## CPU parameters

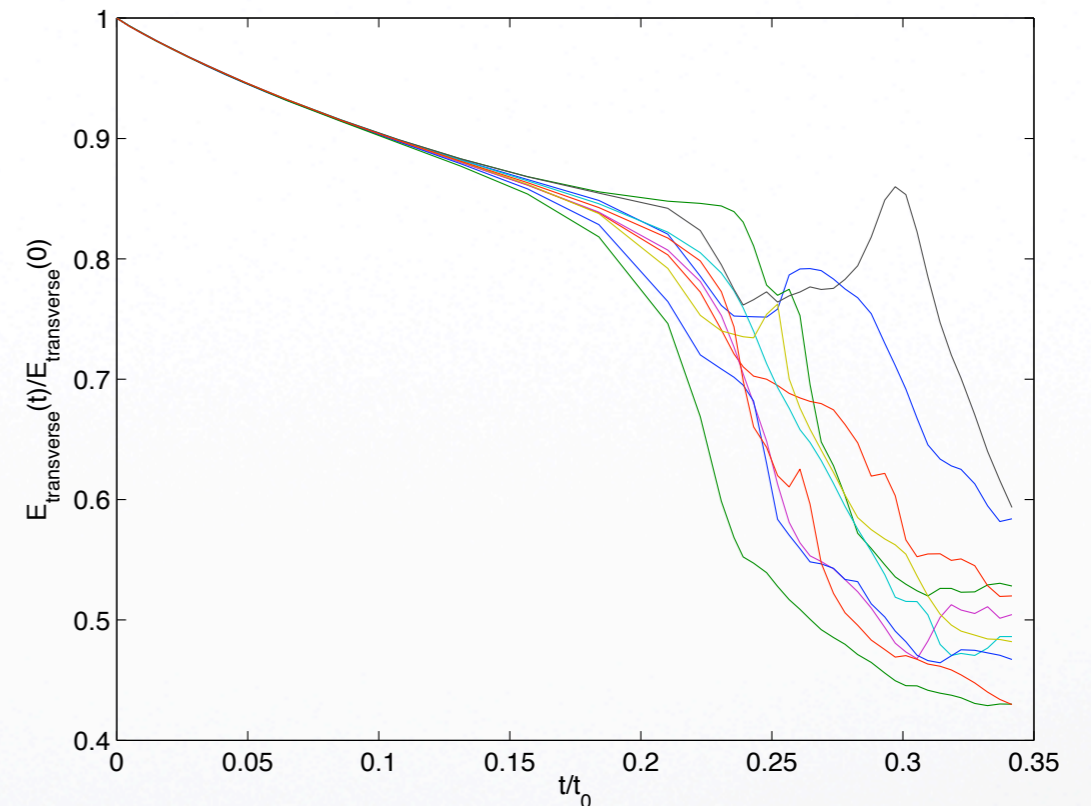
- IBM BlueGene/L
- 4096 CPUs
- 100 hours

# Medium Wavelength Instability

- Spectra



- Cross-flow energy



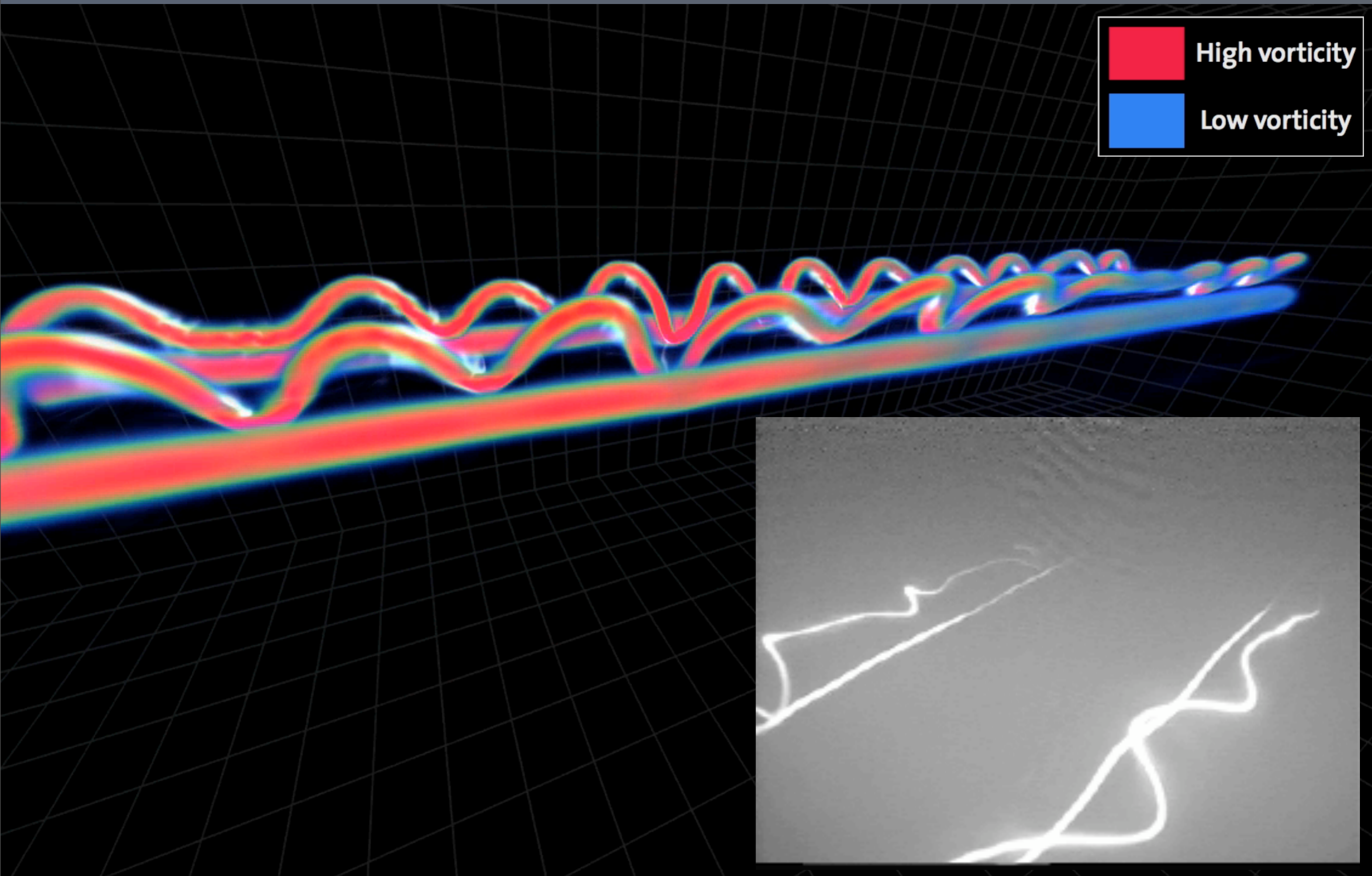
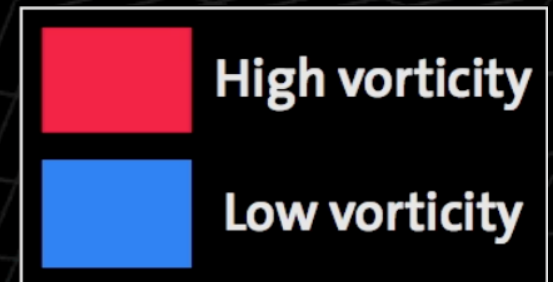
- Mode  $\lambda/b_1 = 0.86 - 0.943$ 
  - Experimental  $\lambda/b_1 = 0.9 - 1.3$
- Bursts when  $\Omega$ -loop feet come together
- Visualization: volume rendering of  $|\omega|$

Rossinelli D. et al., SIGGRAPH08

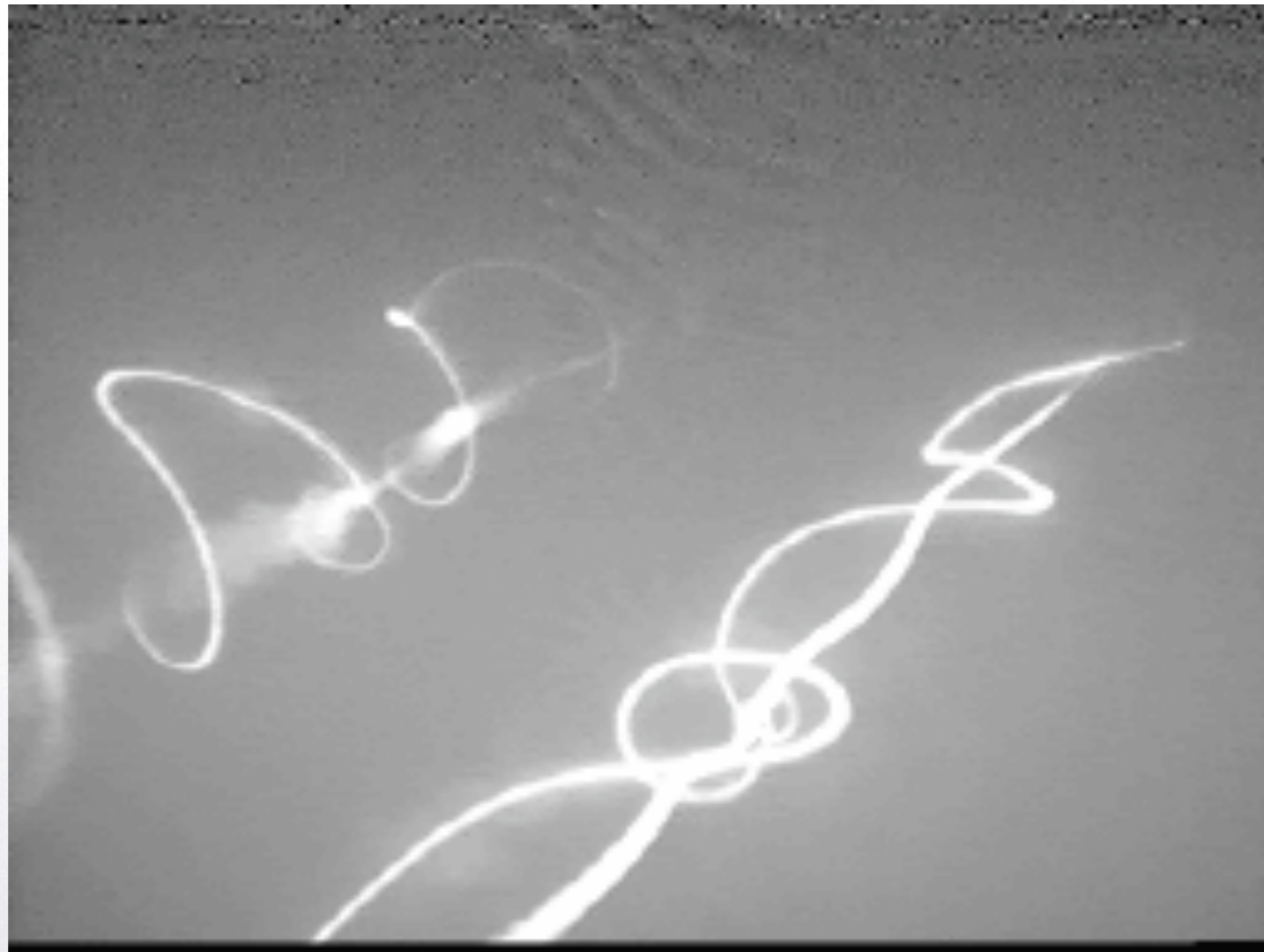


# Instability : Linear phase $t=0.21$

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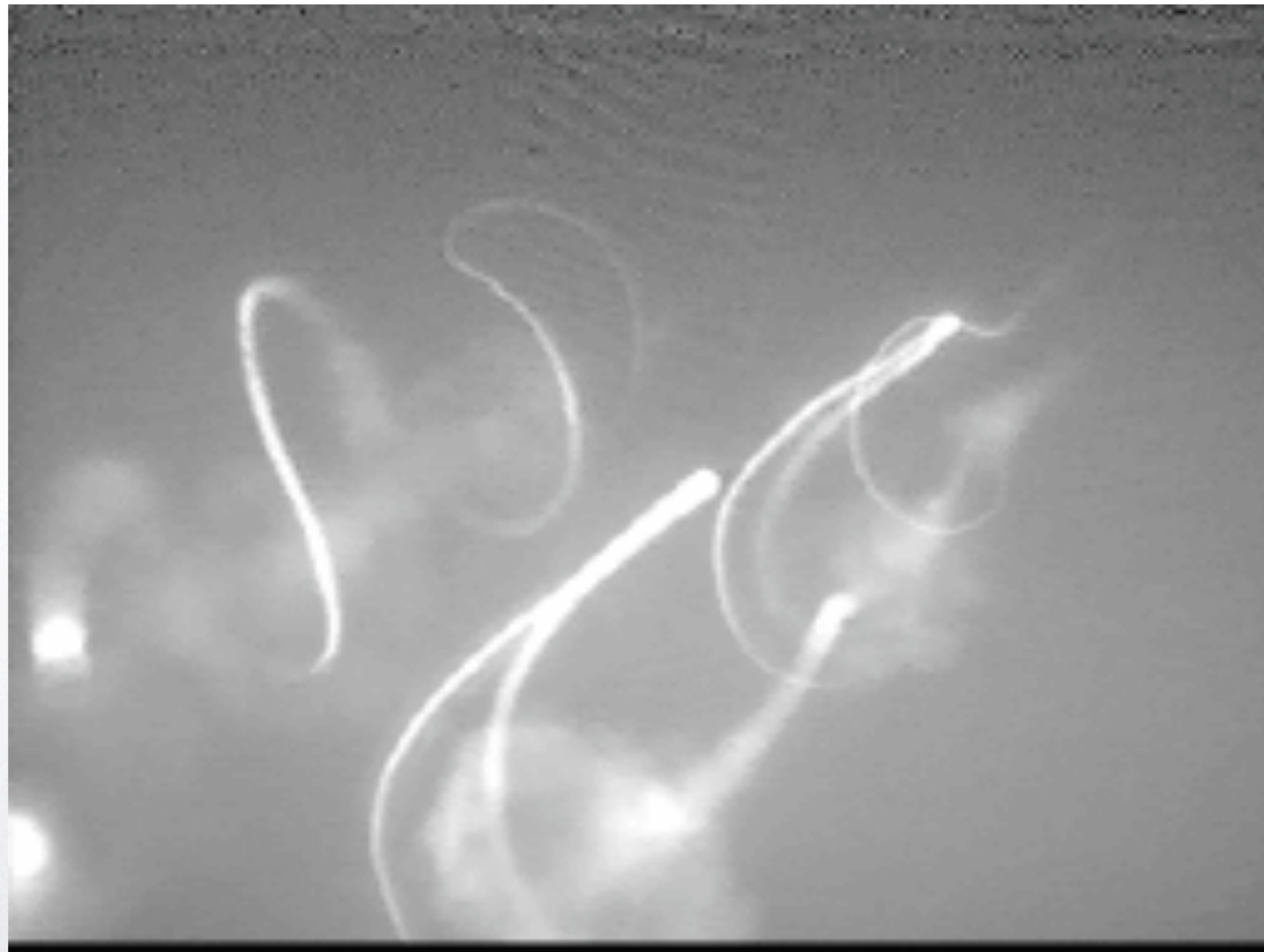


# Instability : **Reconnections** $t=0.25$



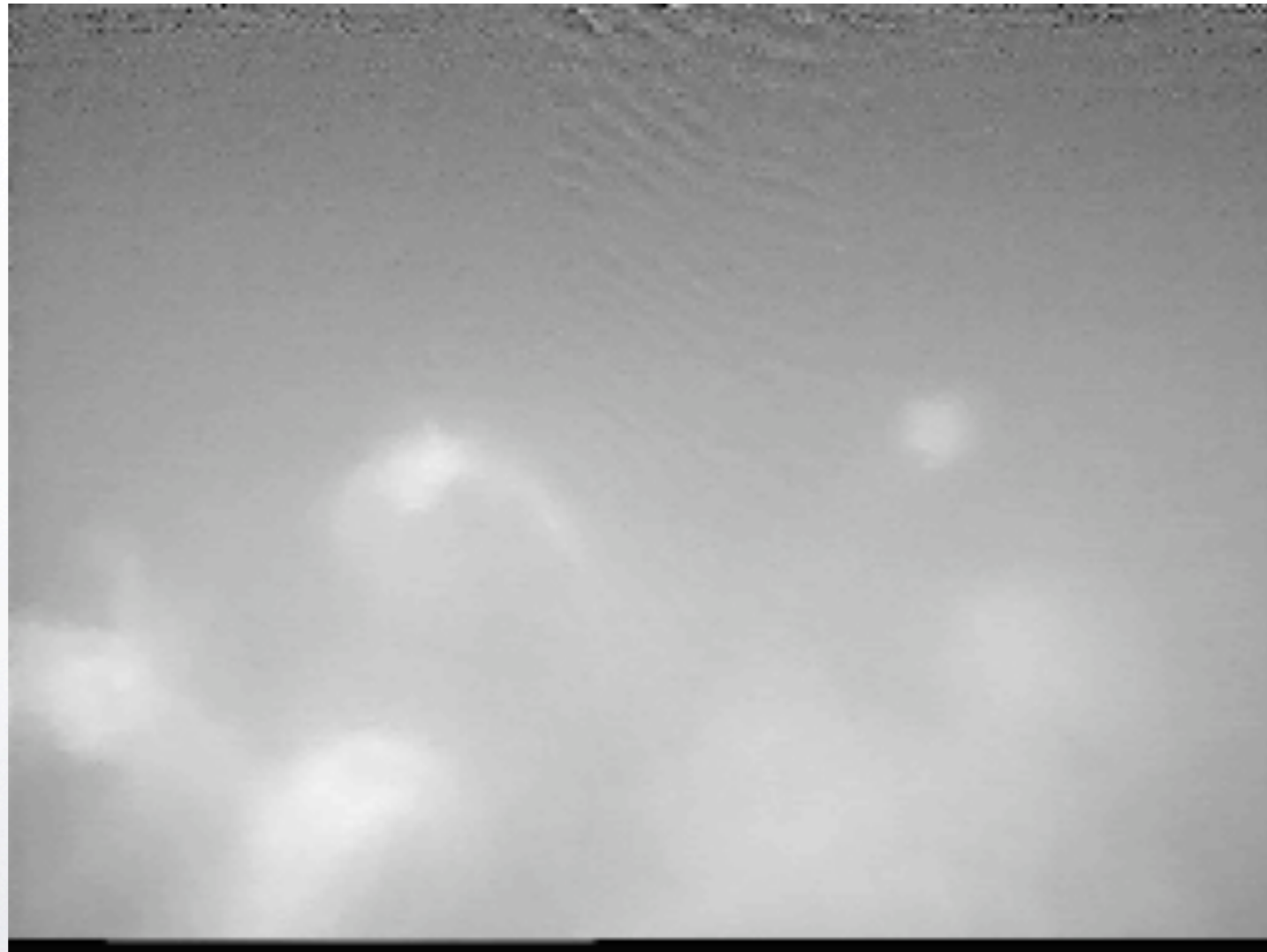
# Instability : **Reconnections** $t=0.25$

# Instability: Propagation $t=0.27$

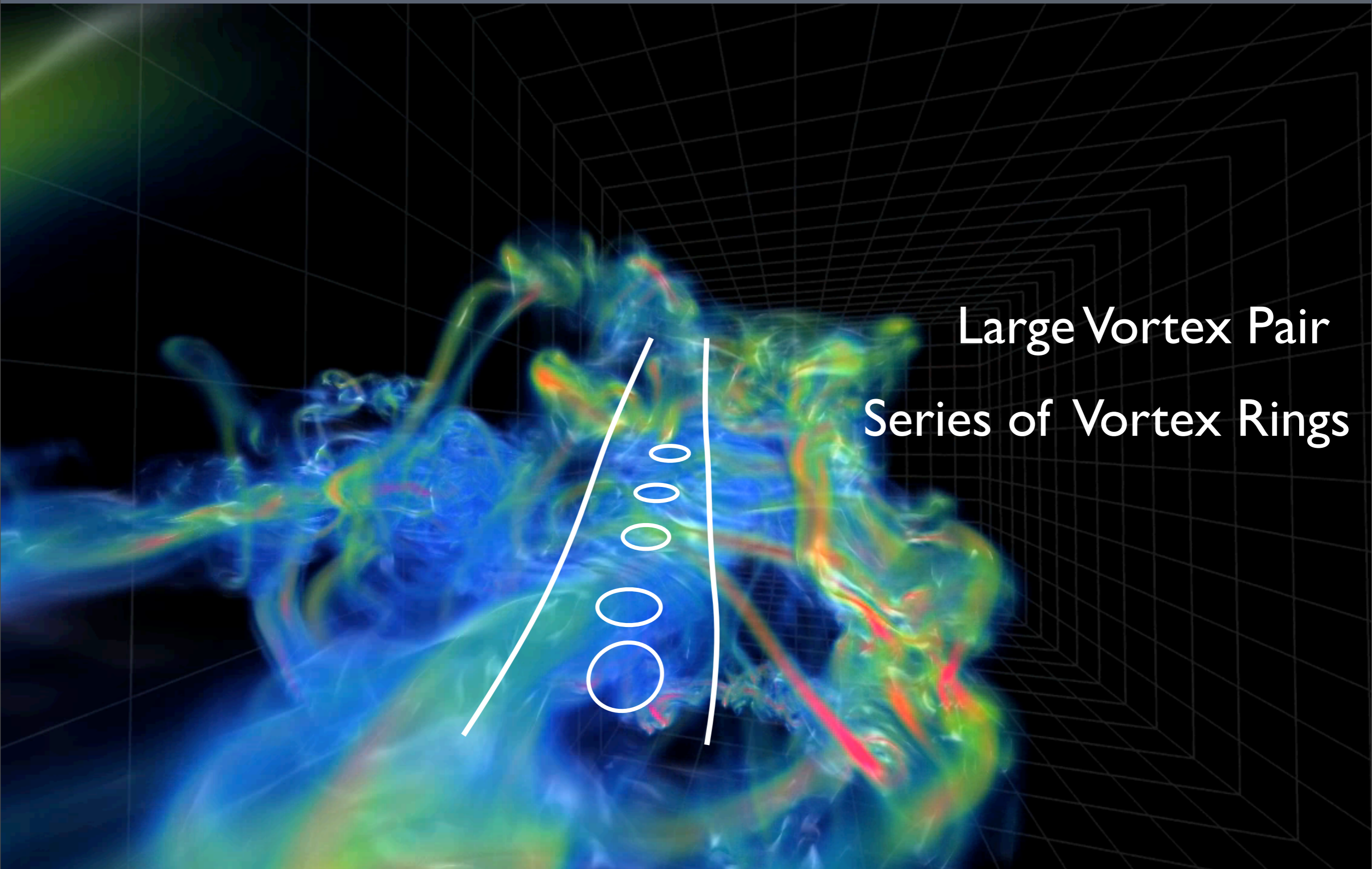


# Instability: Propagation $t=0.27$

# Instability: Decay $t=0.35$



# Instability: **Decay** $t=0.35$



Large Vortex Pair  
Series of Vortex Rings



# Optimization of Vortex Decay

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- Find fastest decaying wake configuration
  - in terms of global end-result: energy, induced rolling moment,...

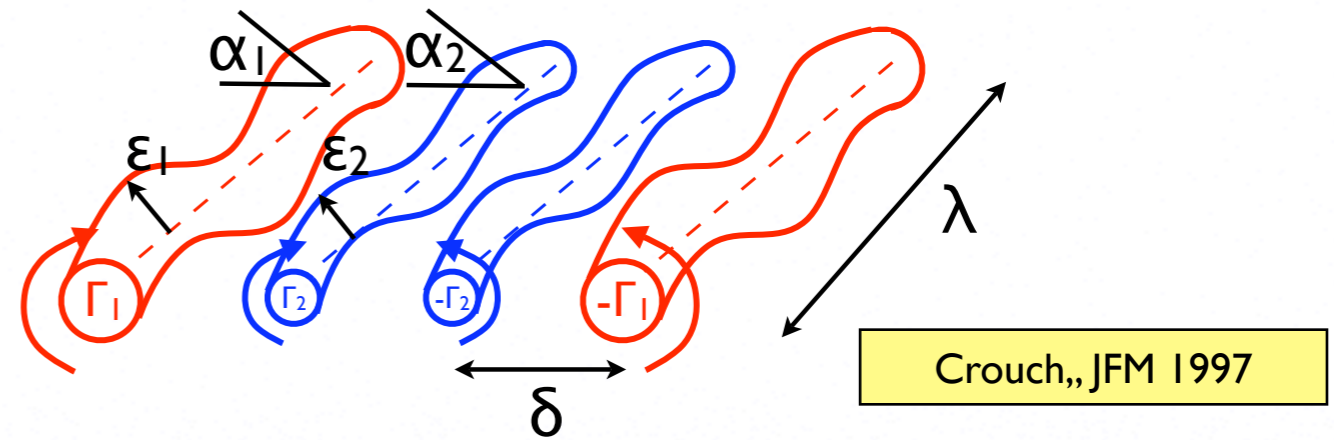
# Optimization of Vortex Decay

- Find fastest decaying wake configuration
  - in terms of global end-result: energy, induced rolling moment,...
- Given, e.g.
  - a range in trim
  - active device perturbations

# Optimization of Vortex Decay

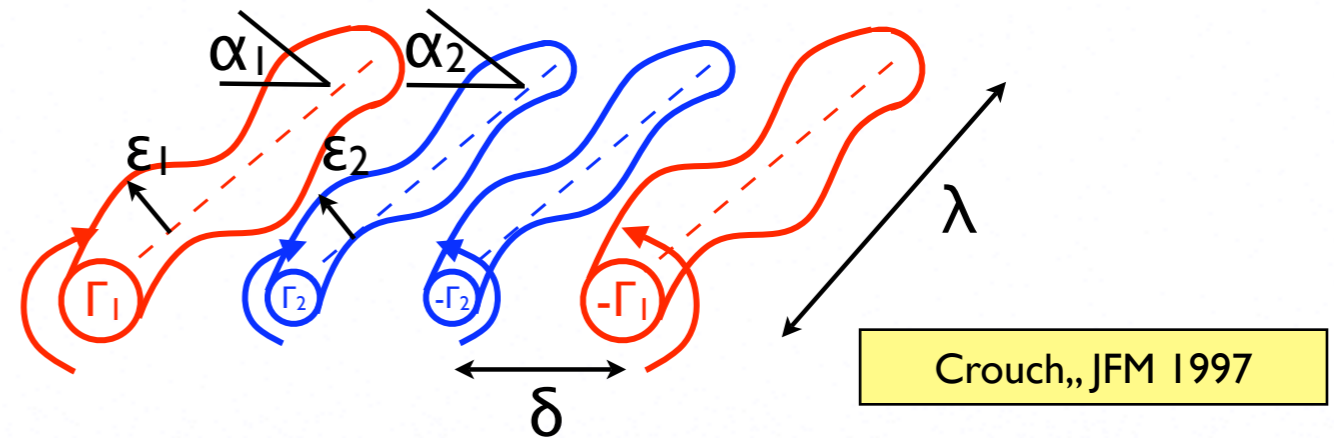
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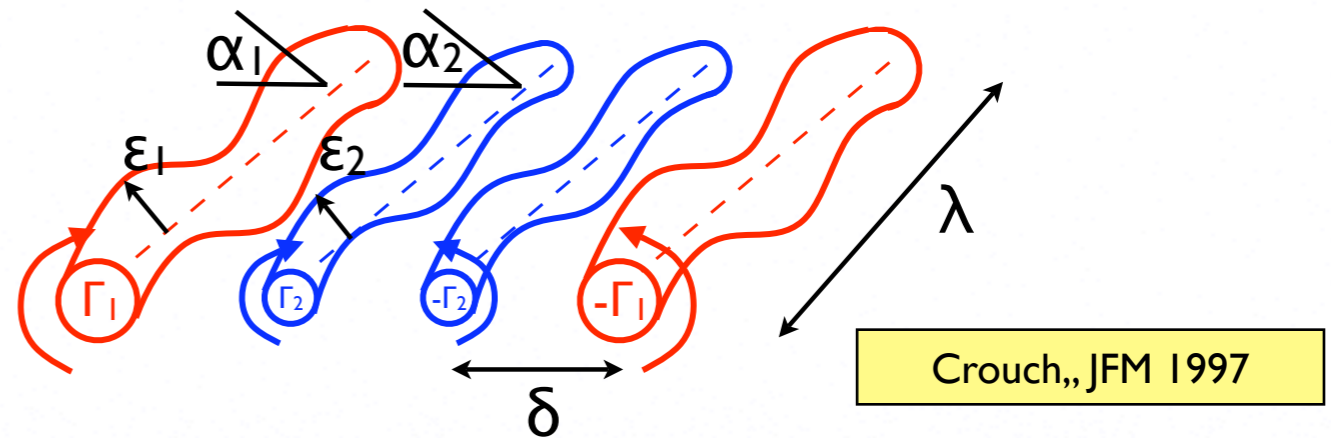


- Problem: minimize **objective function**

$$f_{\text{decay}} = \frac{\int_0^T E(t) dt}{E(0)T} \quad (T = 4)$$

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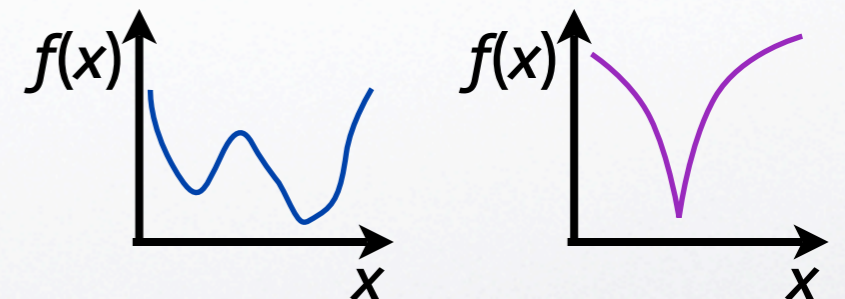


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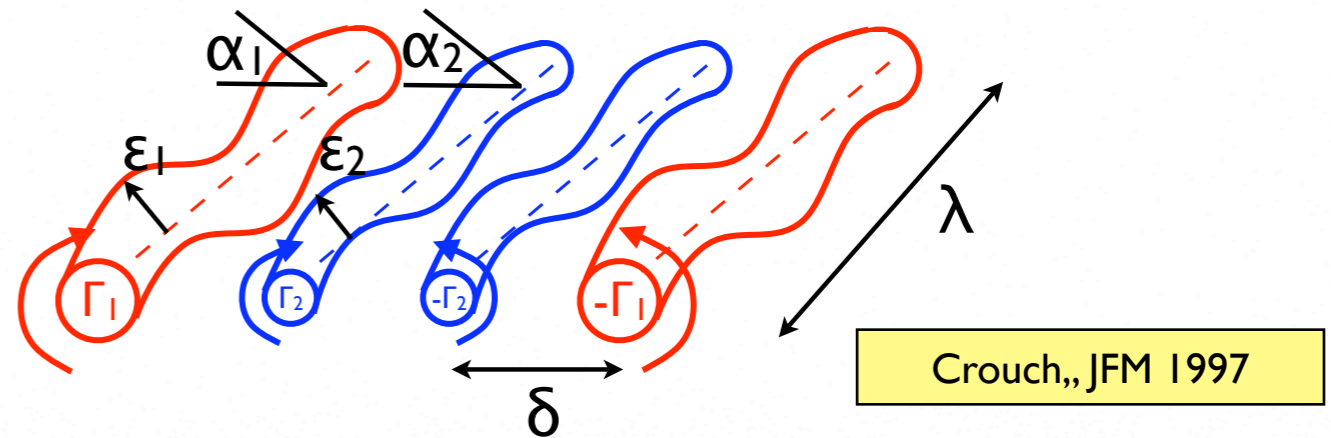
- Challenges

- $f$  can be multi-modal, non-convex, etc.
- $\nabla f$  not readily available
- evaluation of  $f$  is **expensive**



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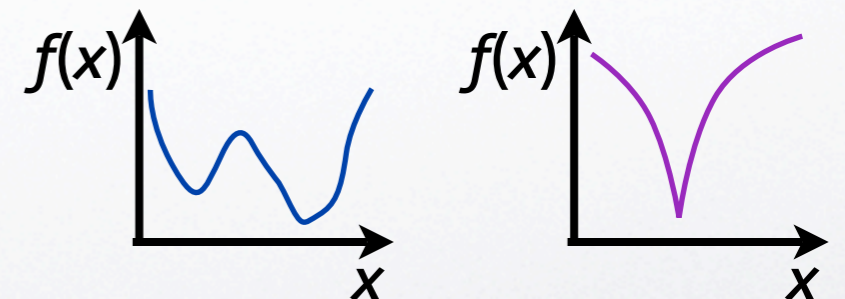


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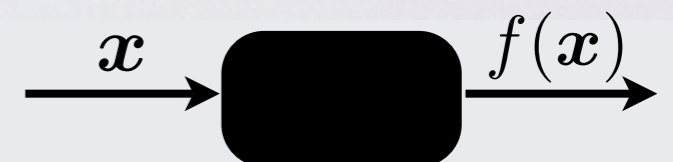
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- Approach

- Evolutionary Optimization
- Prior knowledge encoded in parametrization



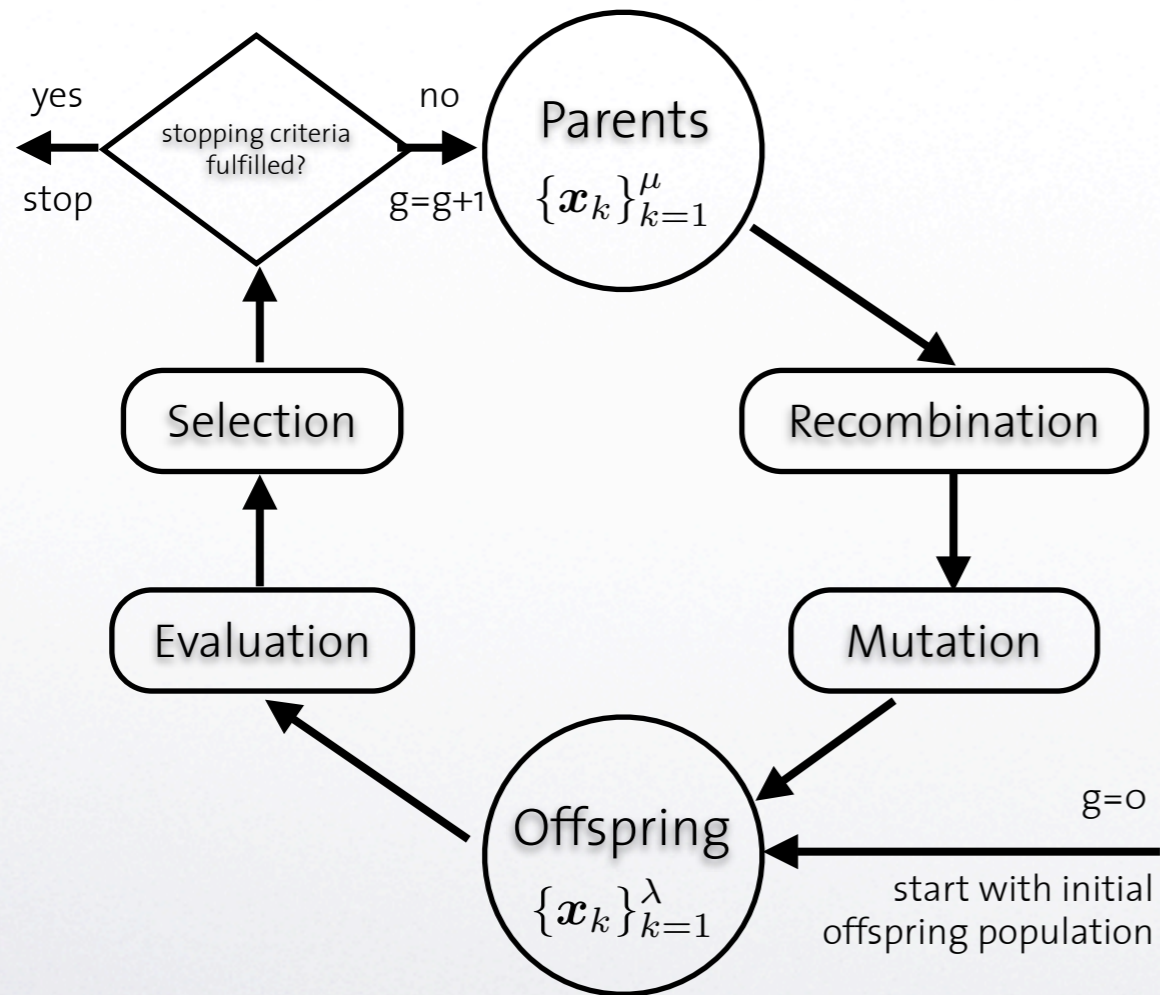


# Optimization of Vortex Decay

## Evolutionary Algorithms (EAs)

- **iterative methods** operating with **populations** of candidate solutions
- Here : Covariance Matrix Adaptation - ES

Hansen et al., Evol. Comput. 2003

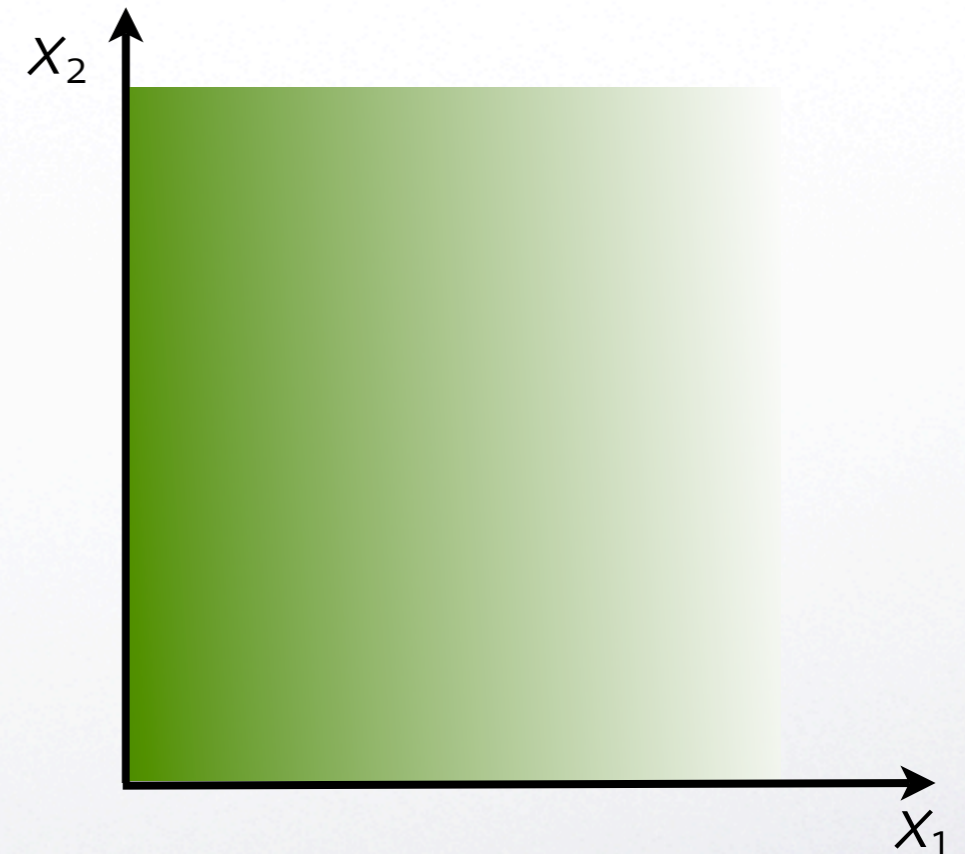
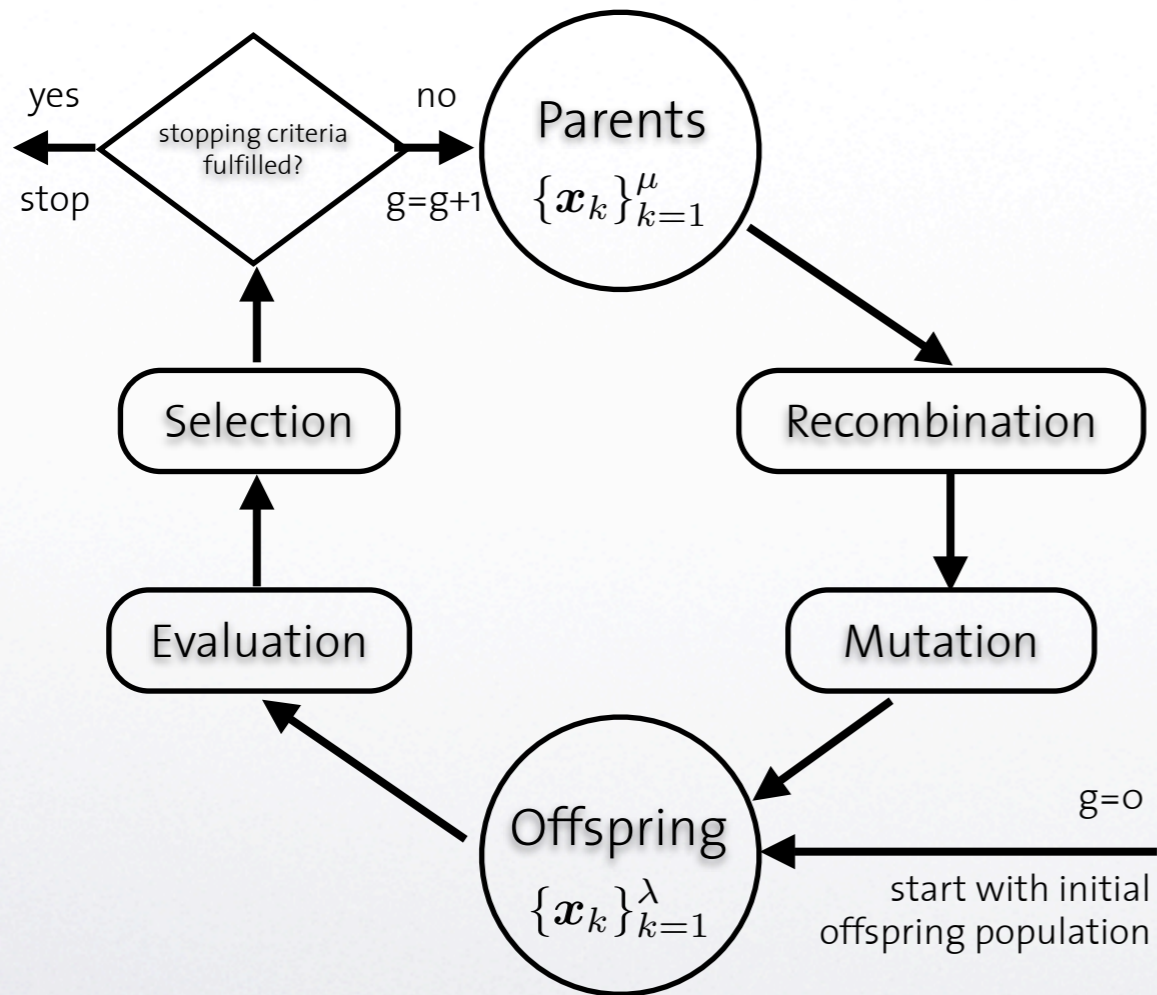


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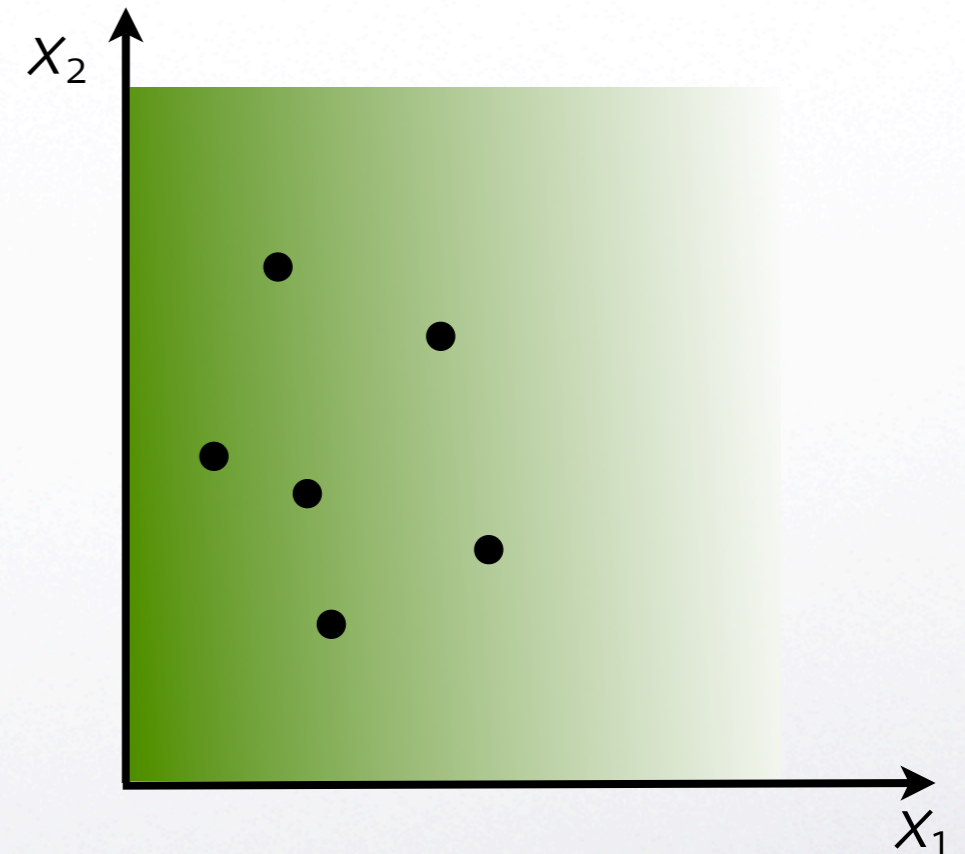
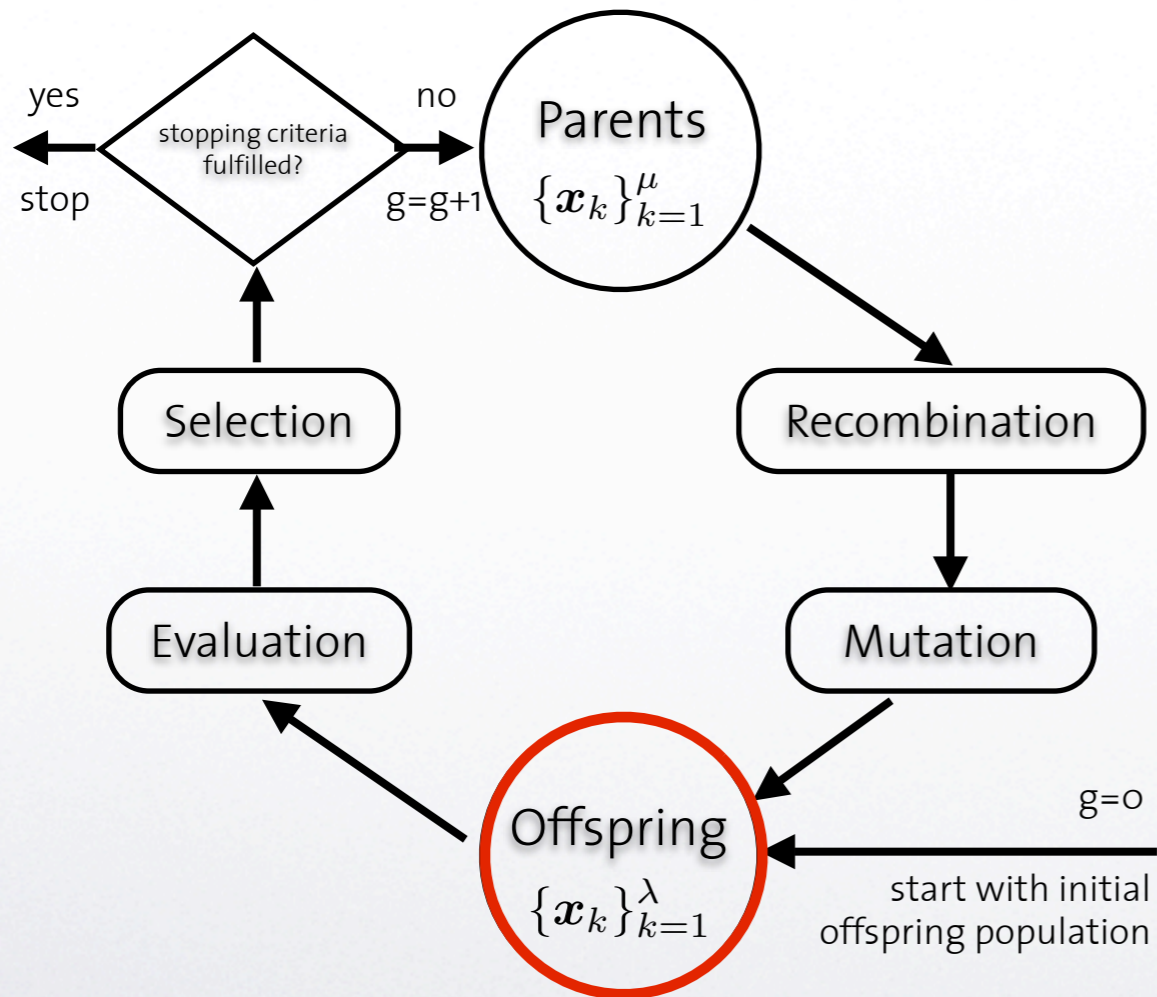


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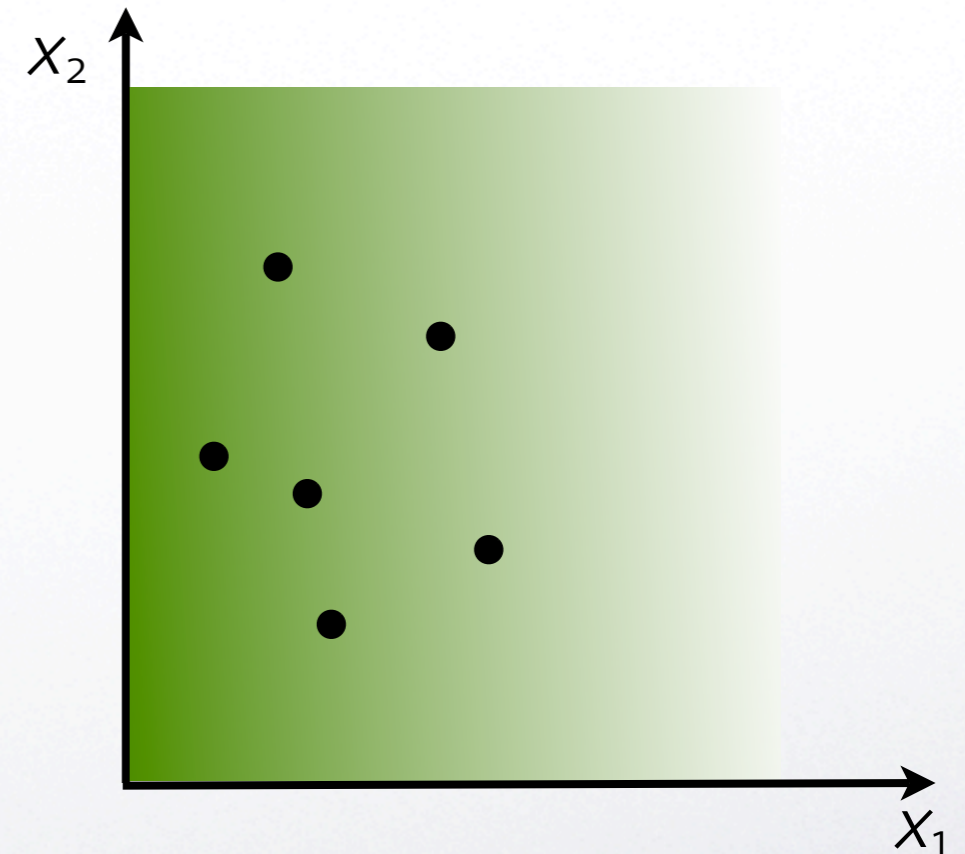
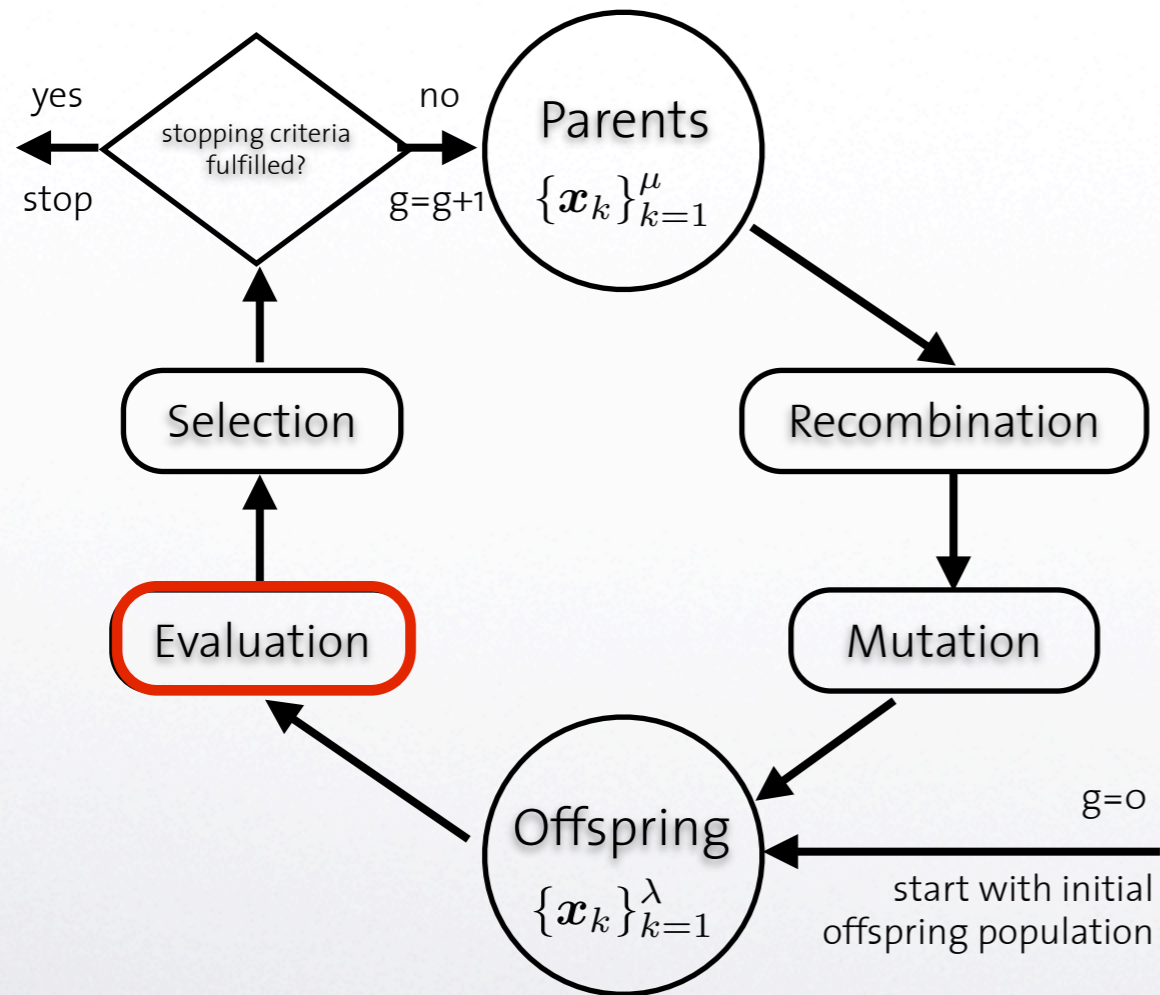


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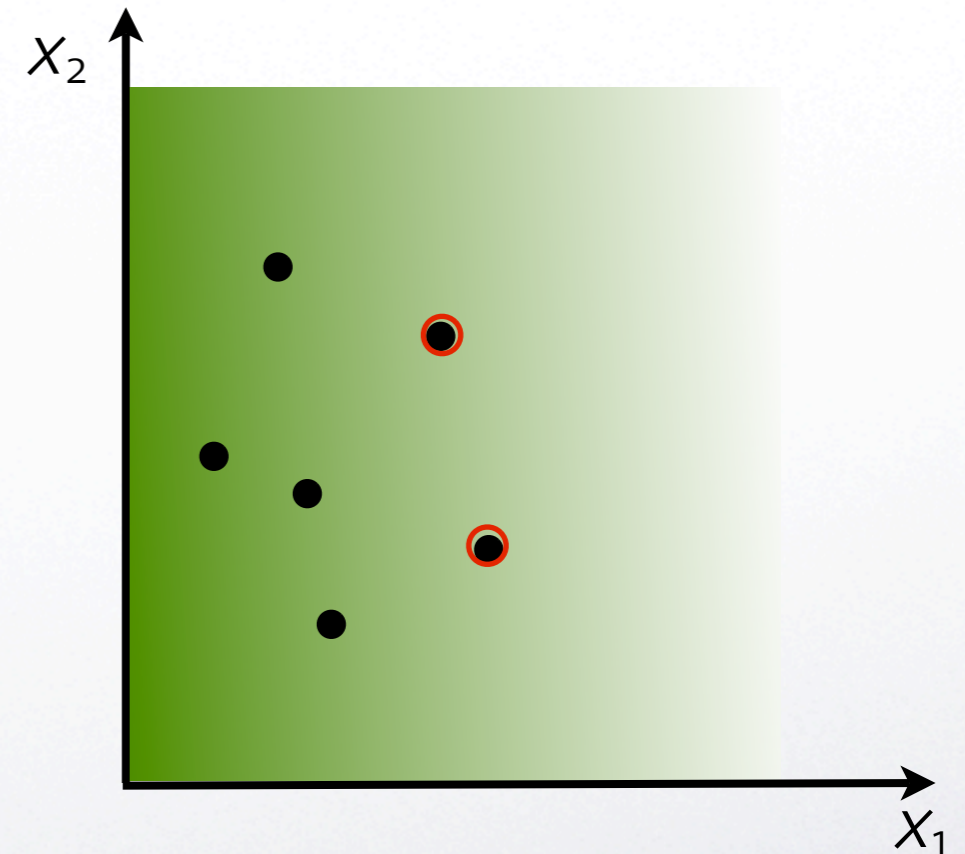
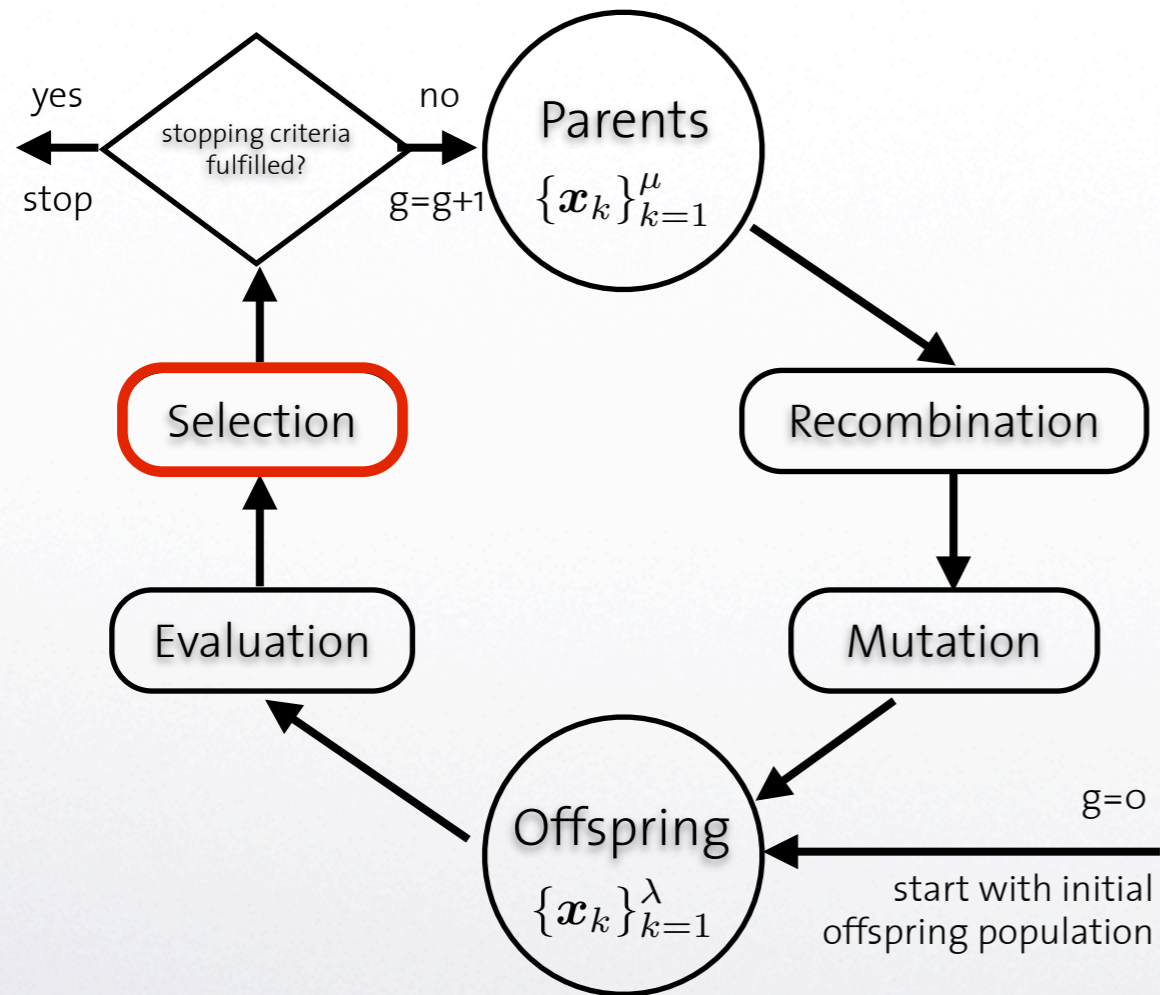


# Optimization of Vortex Decay

## Evolutionary Algorithms (EAs)

- **iterative methods** operating with **populations** of candidate solutions
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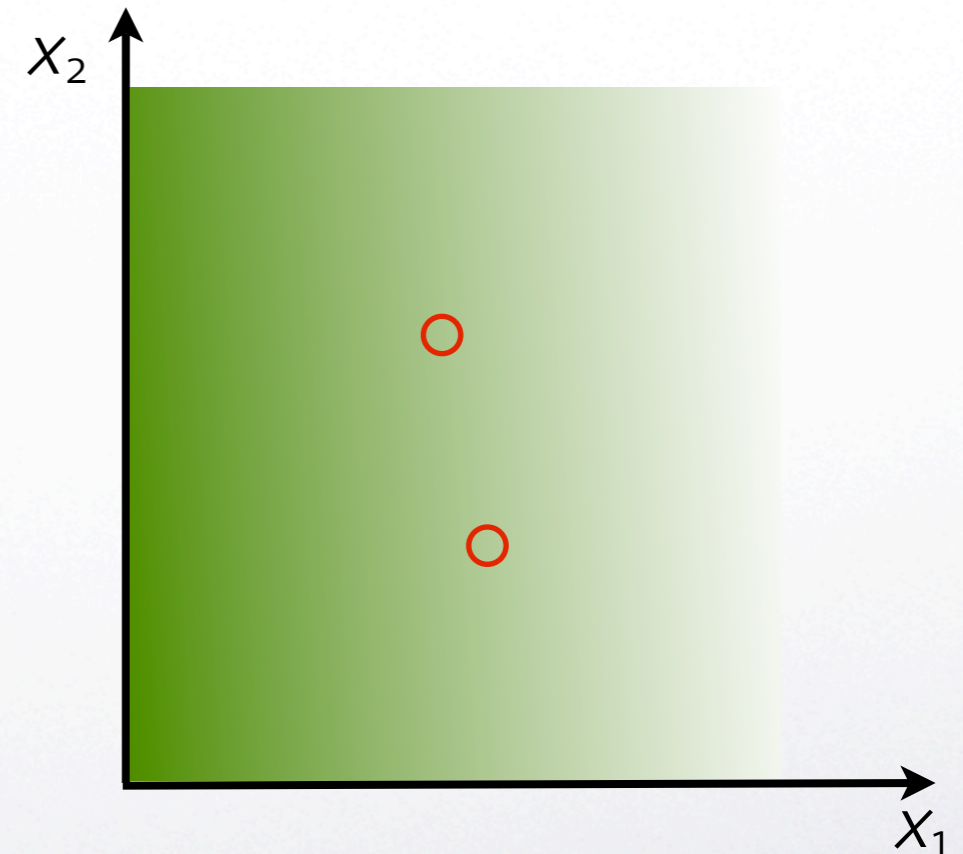
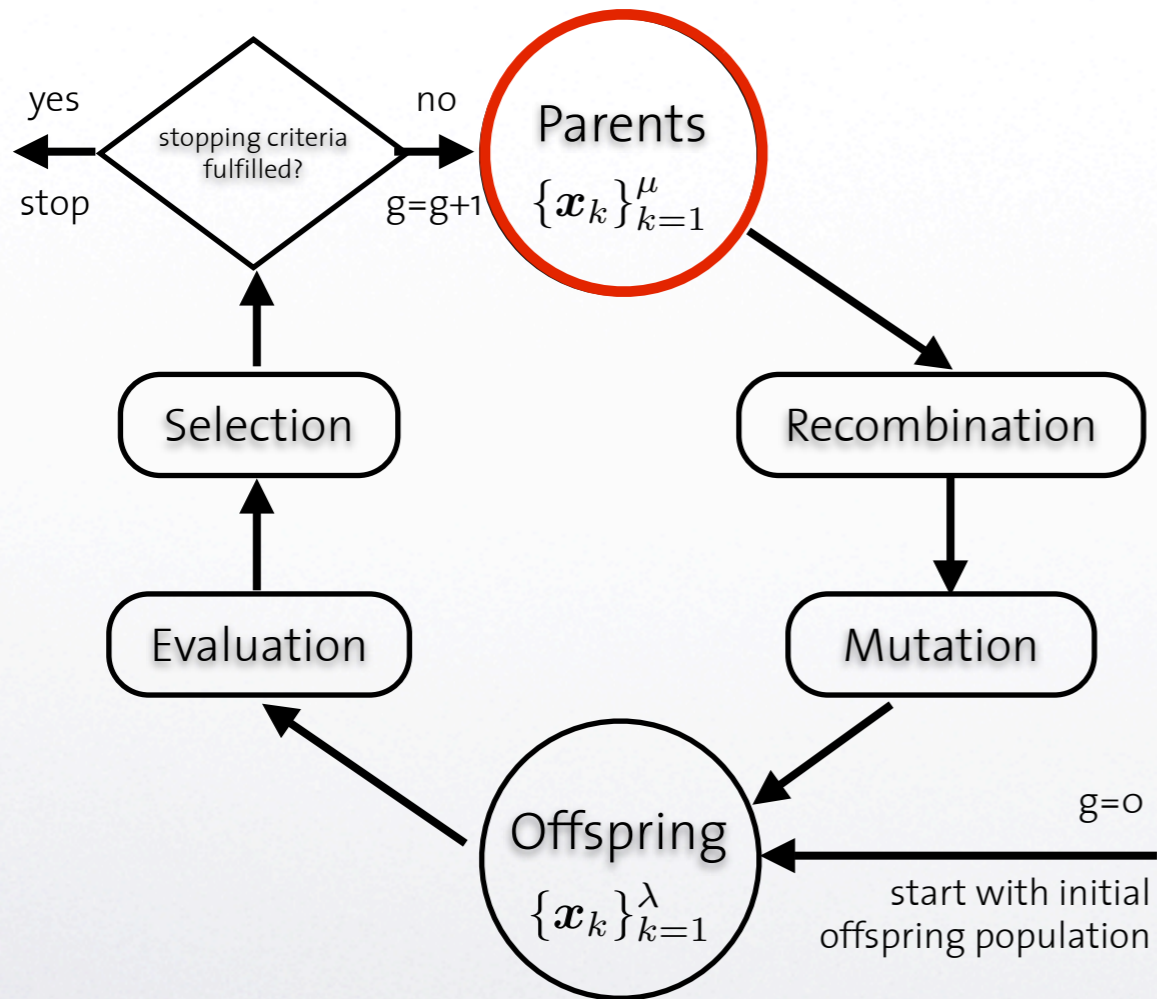


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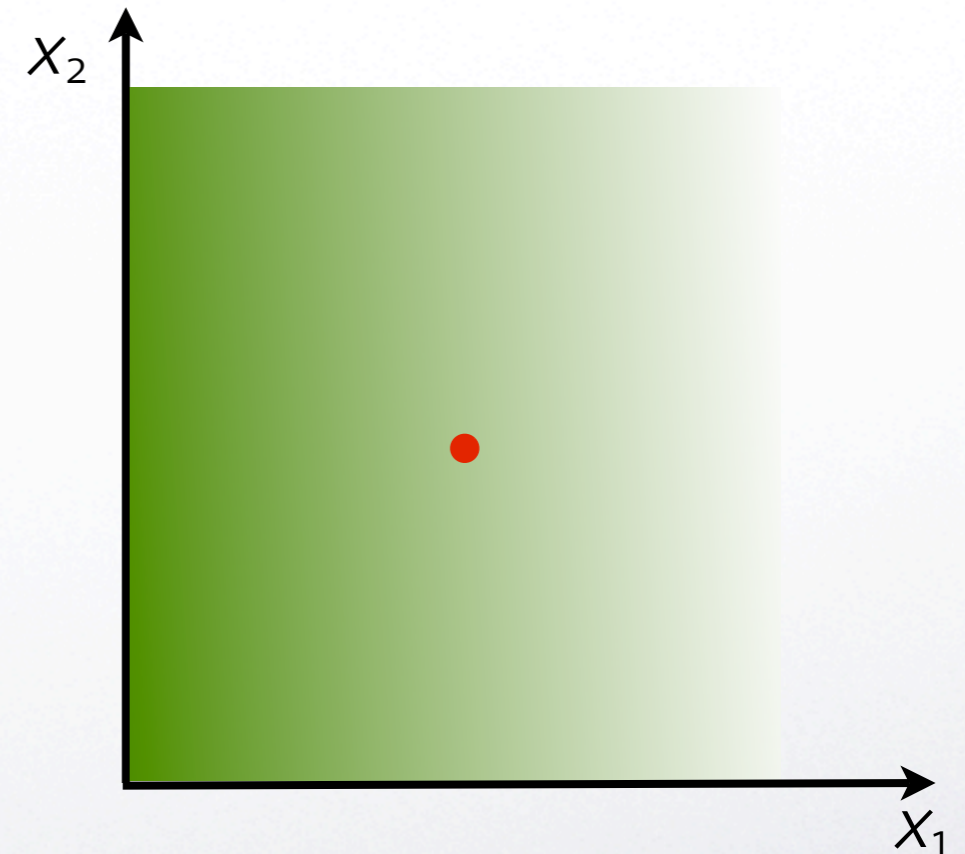
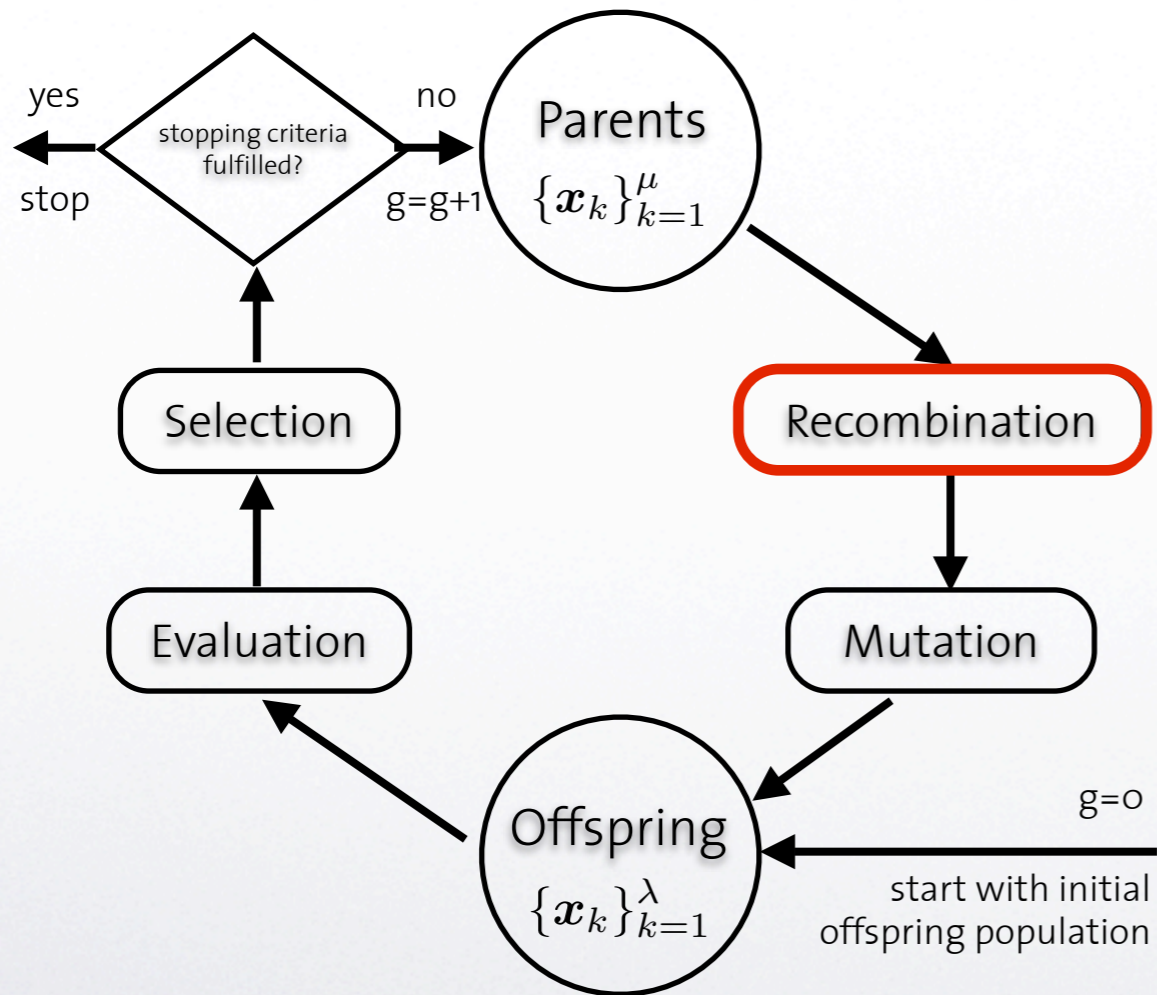


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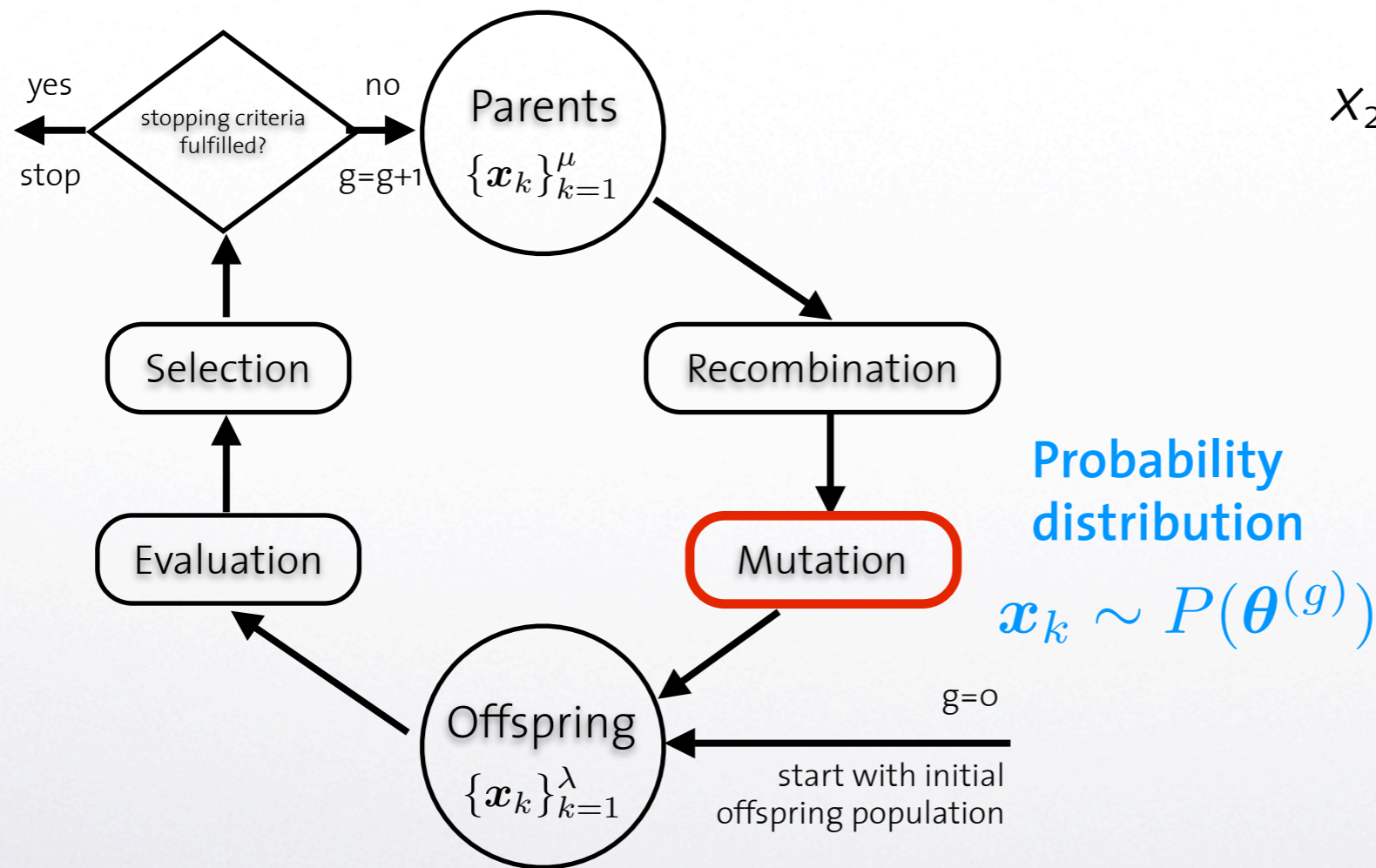


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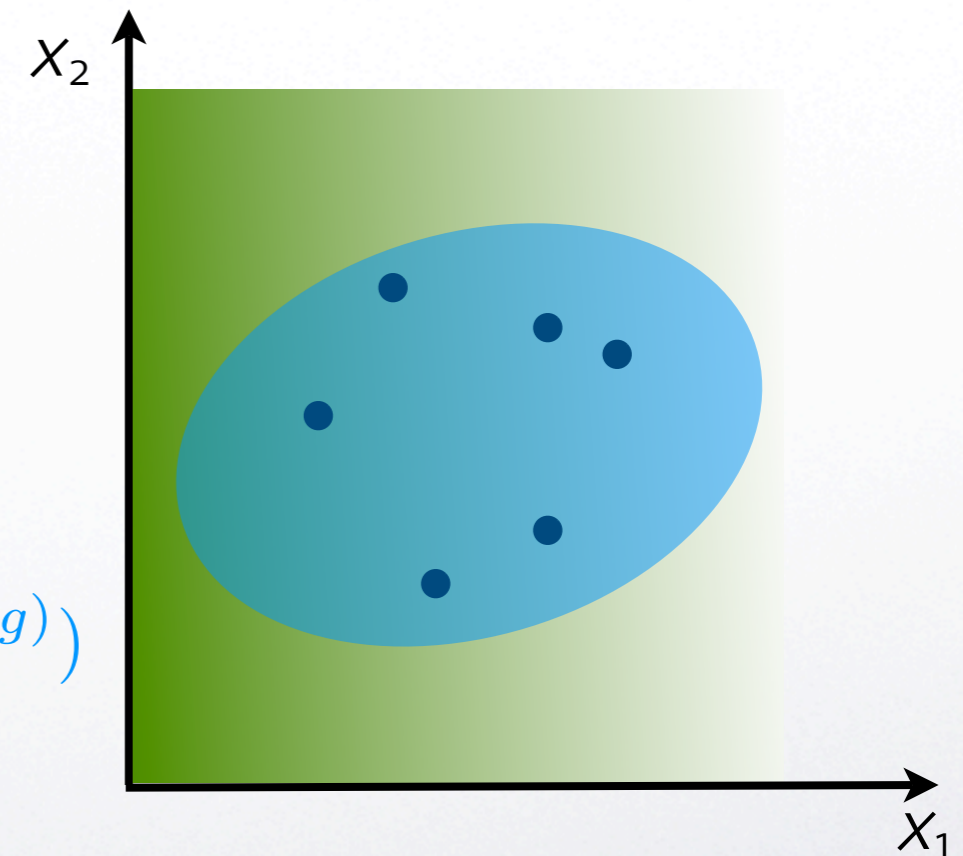
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Probability  
distribution  
 $\mathbf{x}_k \sim P(\boldsymbol{\theta}^{(g)})$



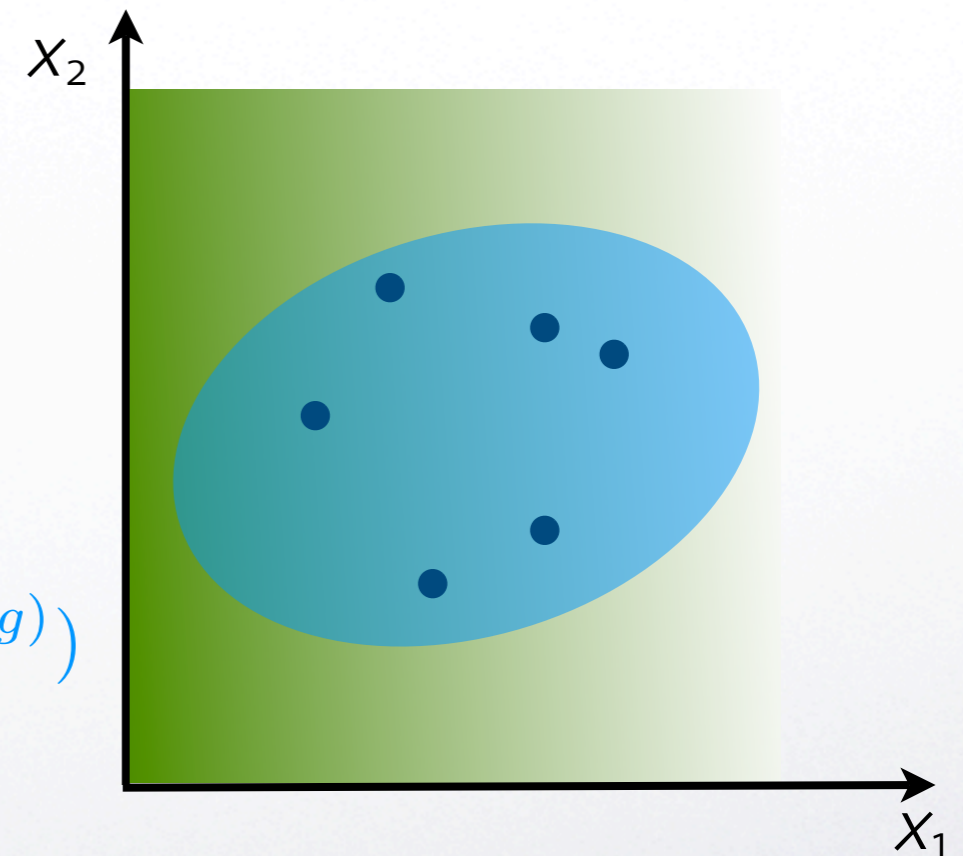
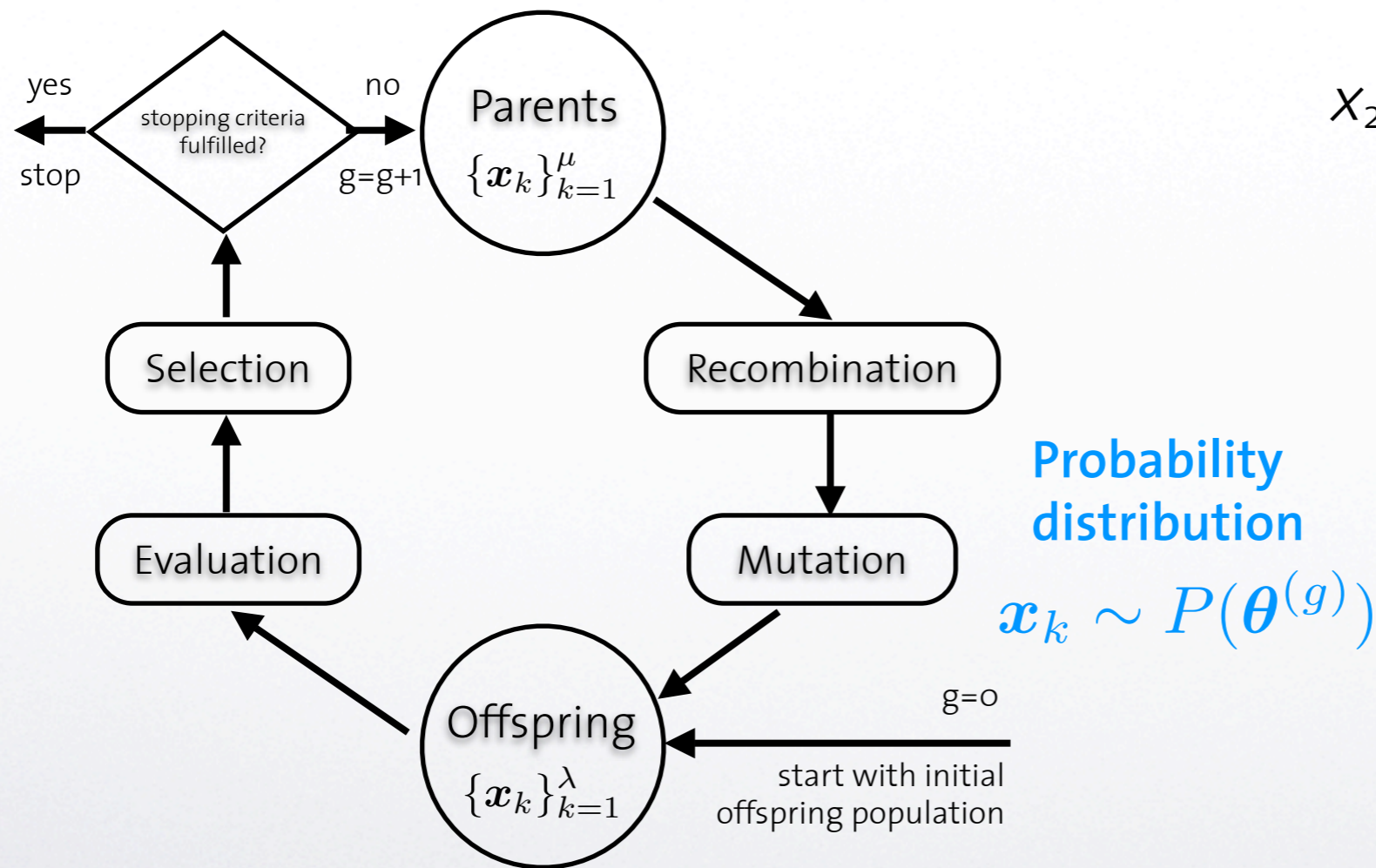


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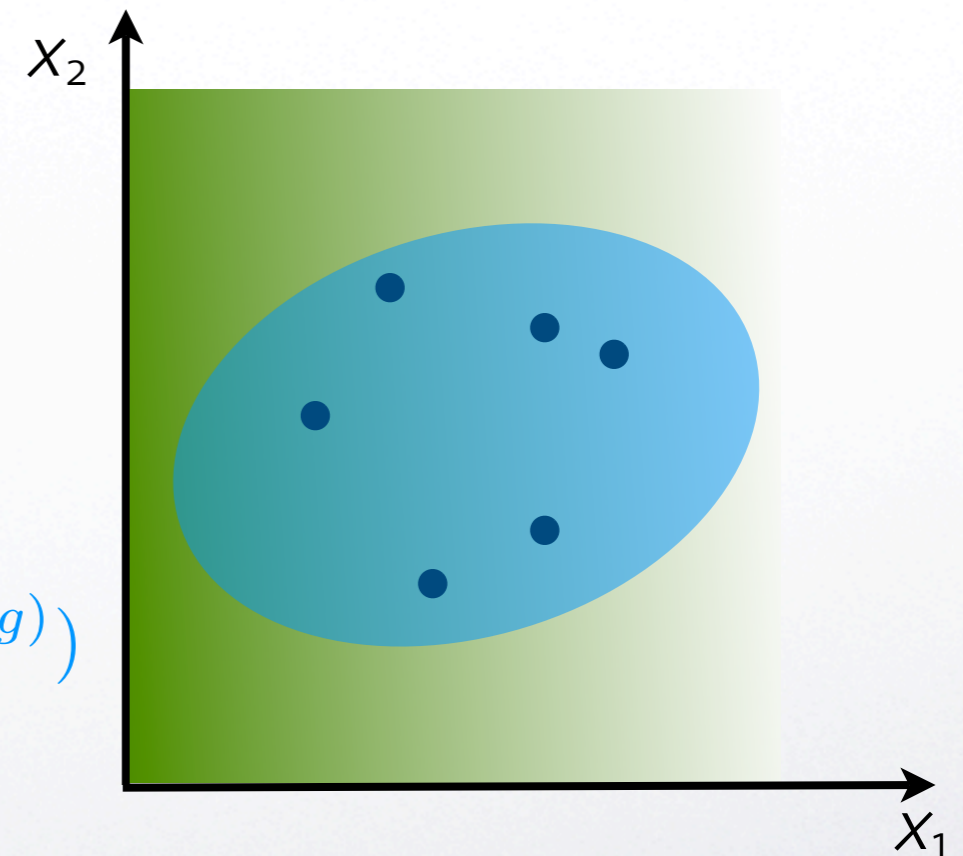
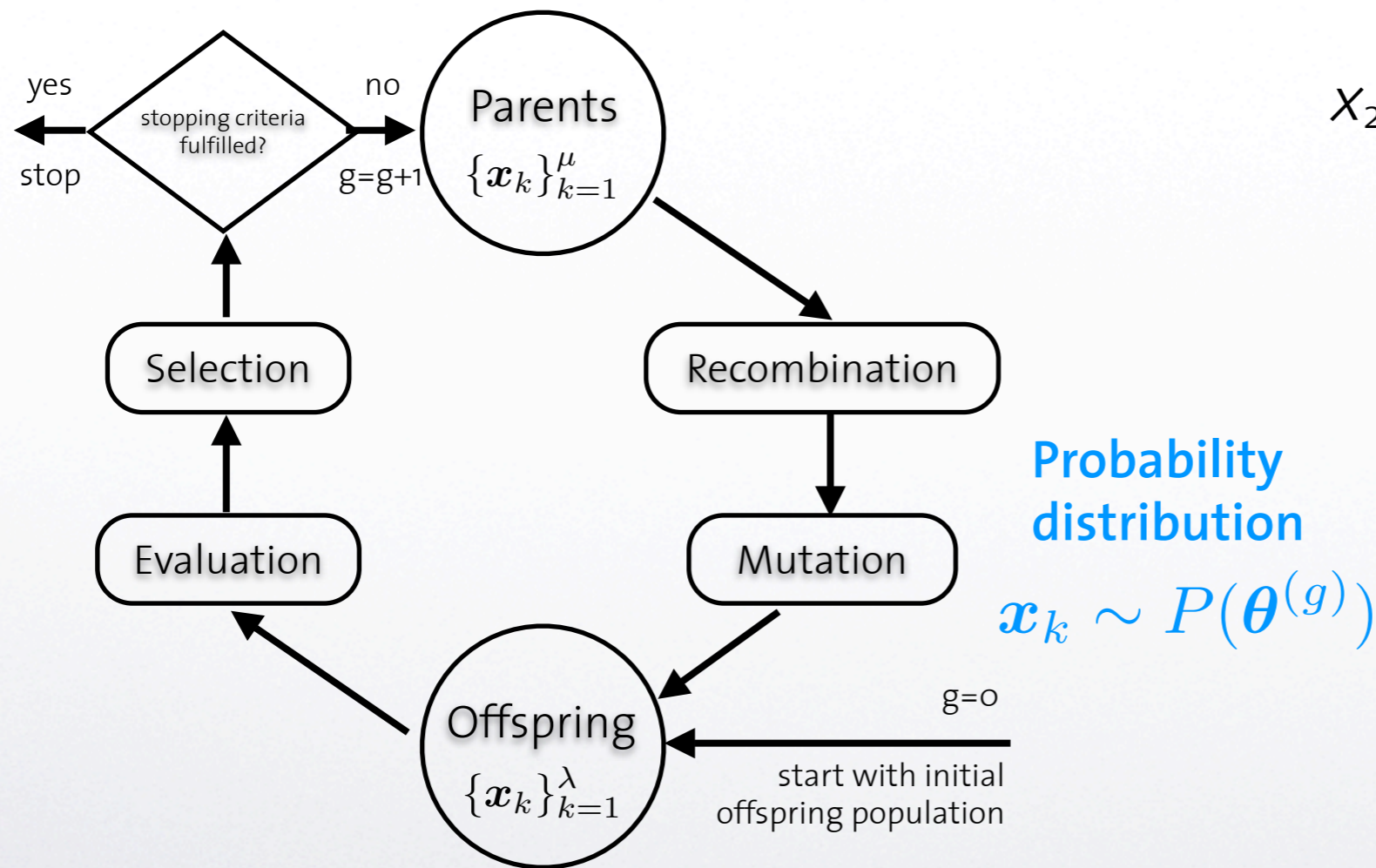
- popular because of their flexibility and **robustness**

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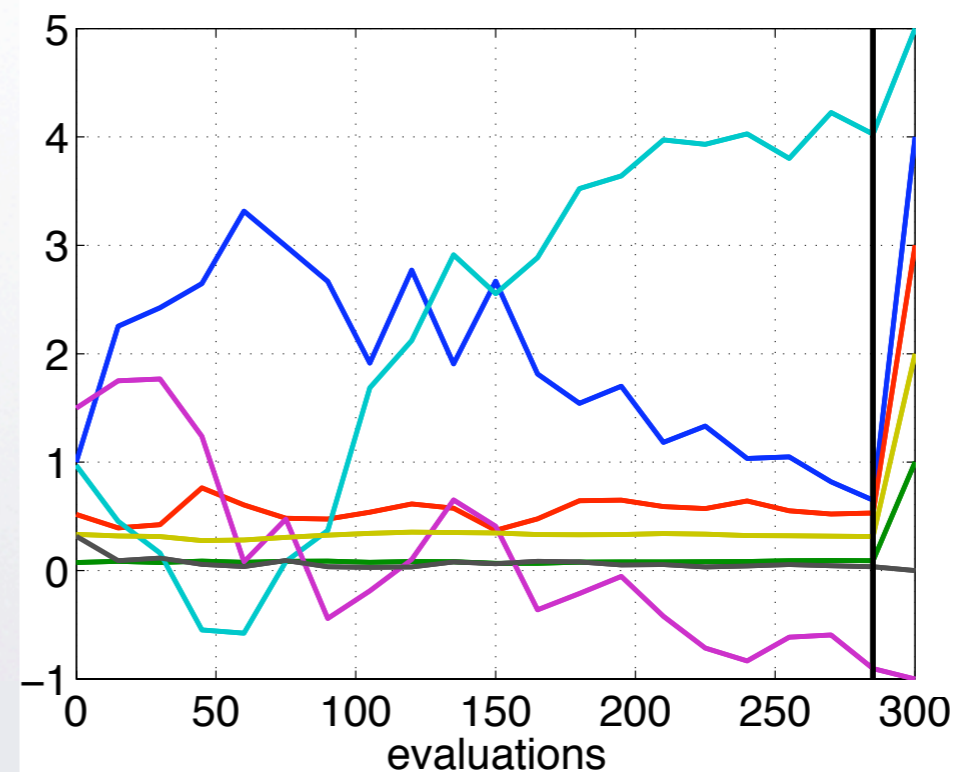
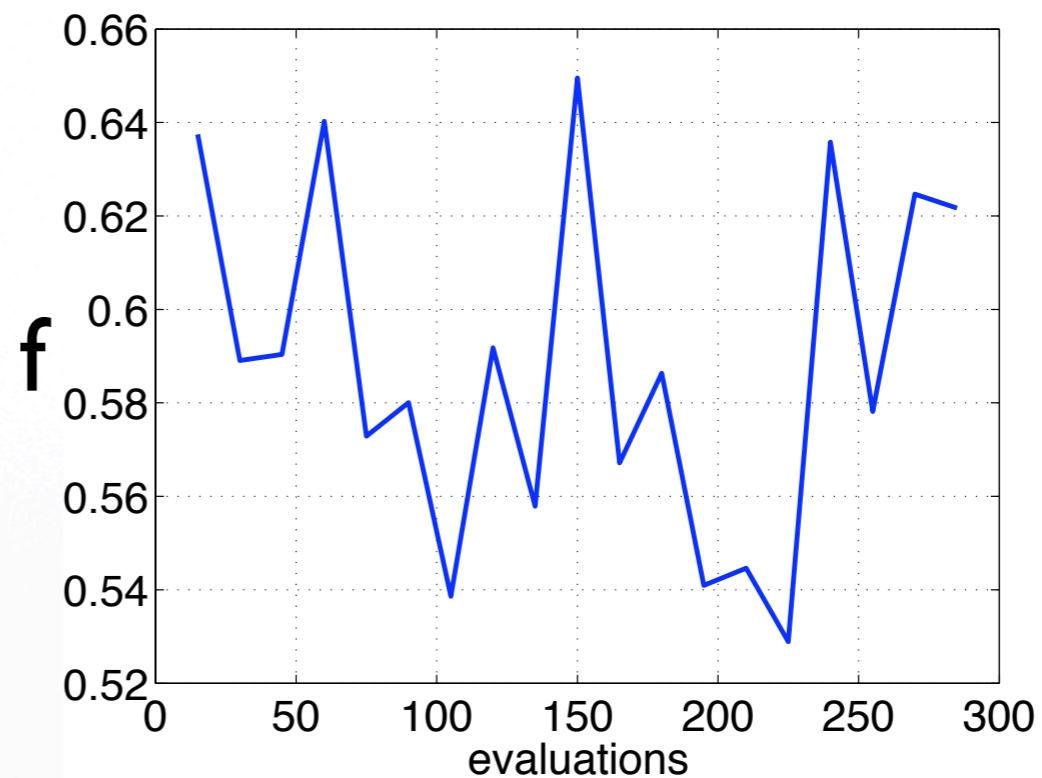
- popular because of their flexibility and **robustness**
- main **disadvantage**: Need **large number of objective function evaluations**

# Optimization of Vortex Decay

**f**

# Optimization of Vortex Decay

- PRESENT
  - After sweep toward large  $\lambda$ , moves toward smaller values
  - Convergence



$\alpha_1$

$\lambda$

$\arctan(\epsilon_2, \epsilon_1)$

$\delta$

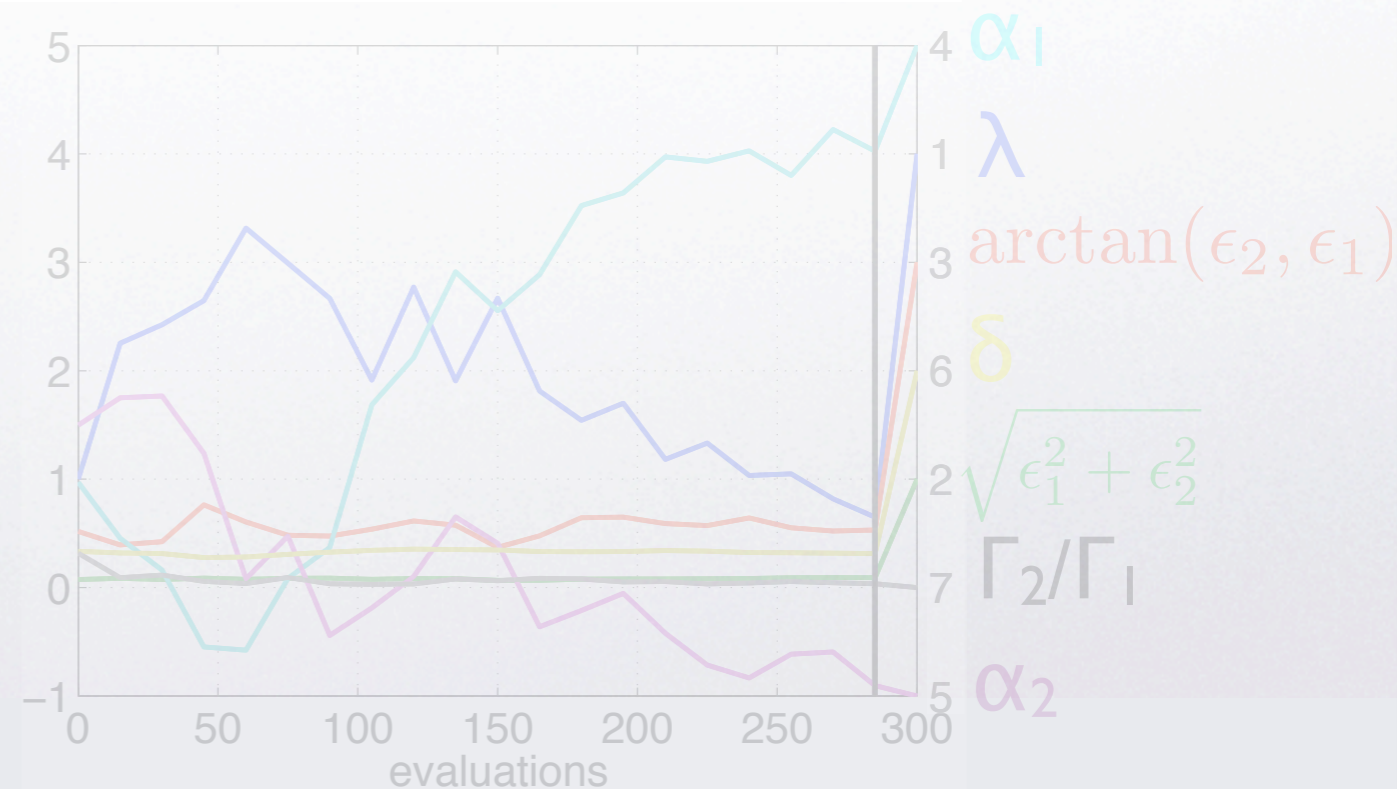
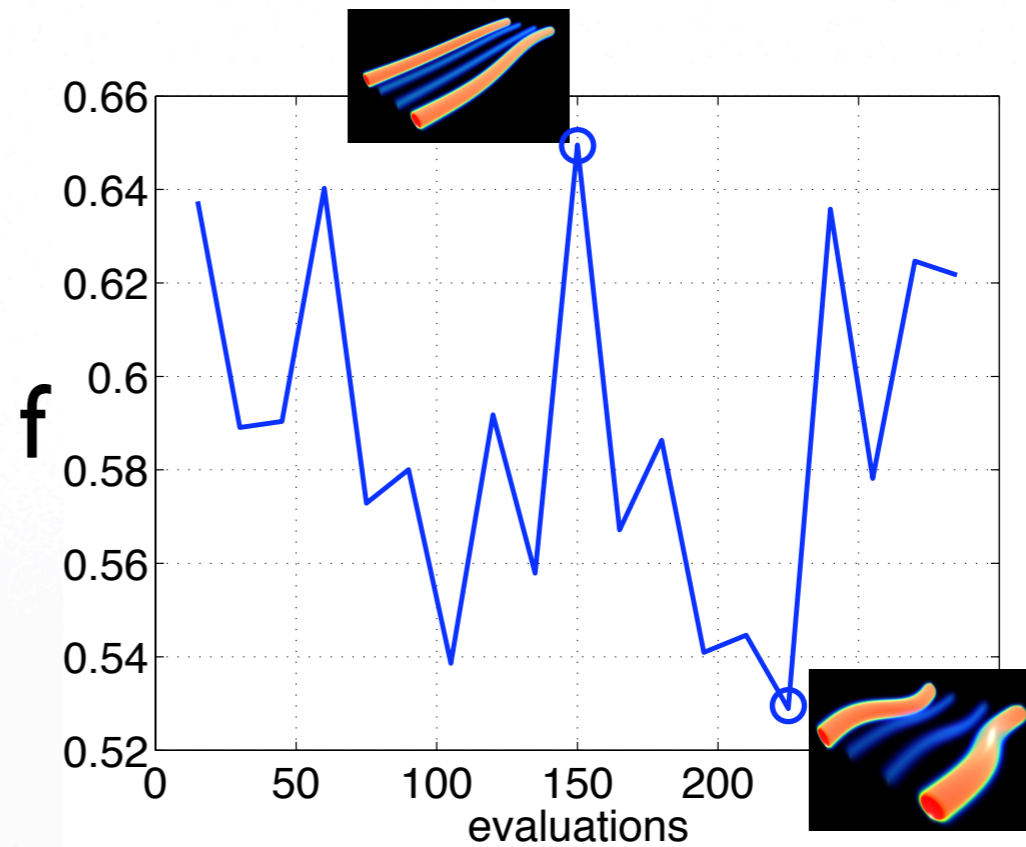
$\sqrt{\epsilon_1^2 + \epsilon_2^2}$

$\Gamma_2/\Gamma_1$

$\alpha_2$

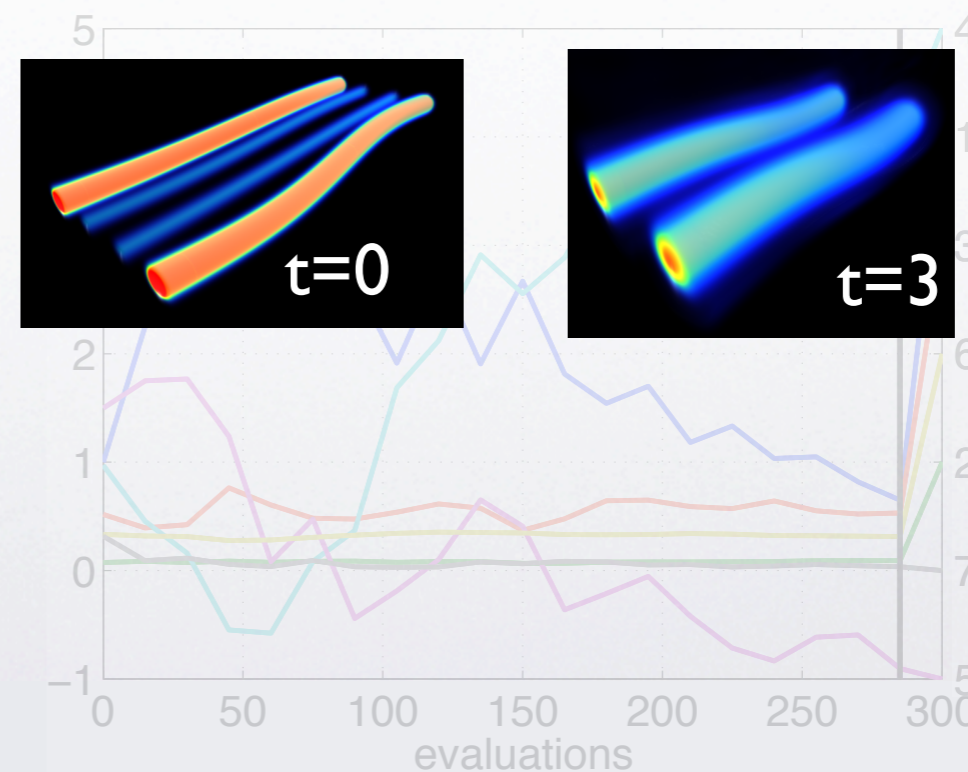
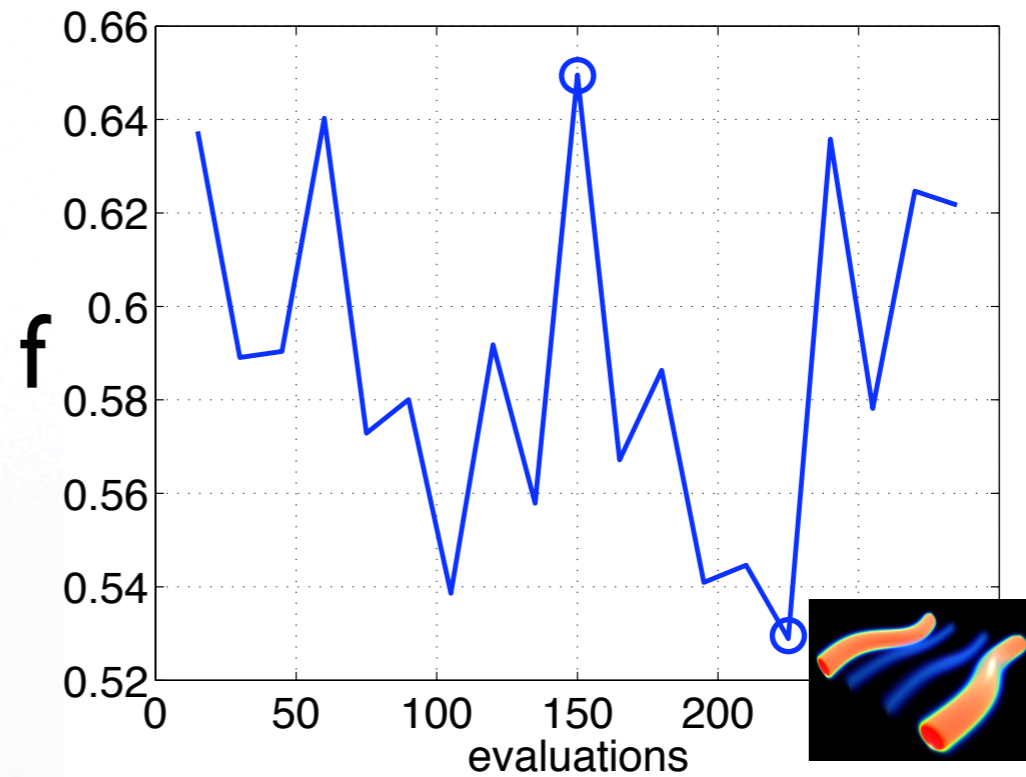
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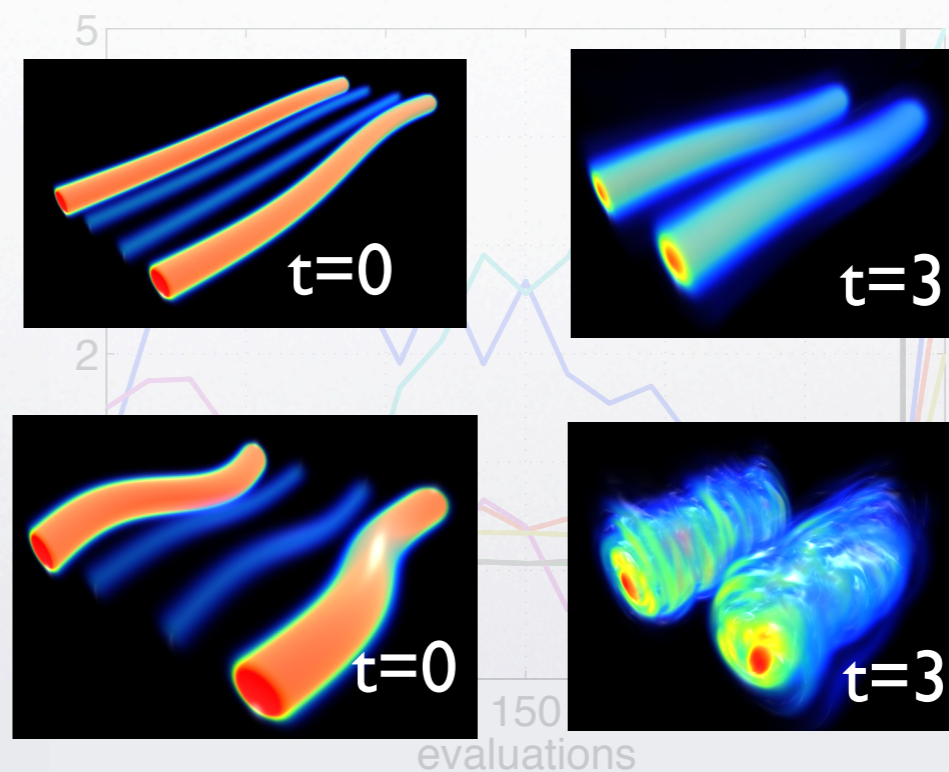
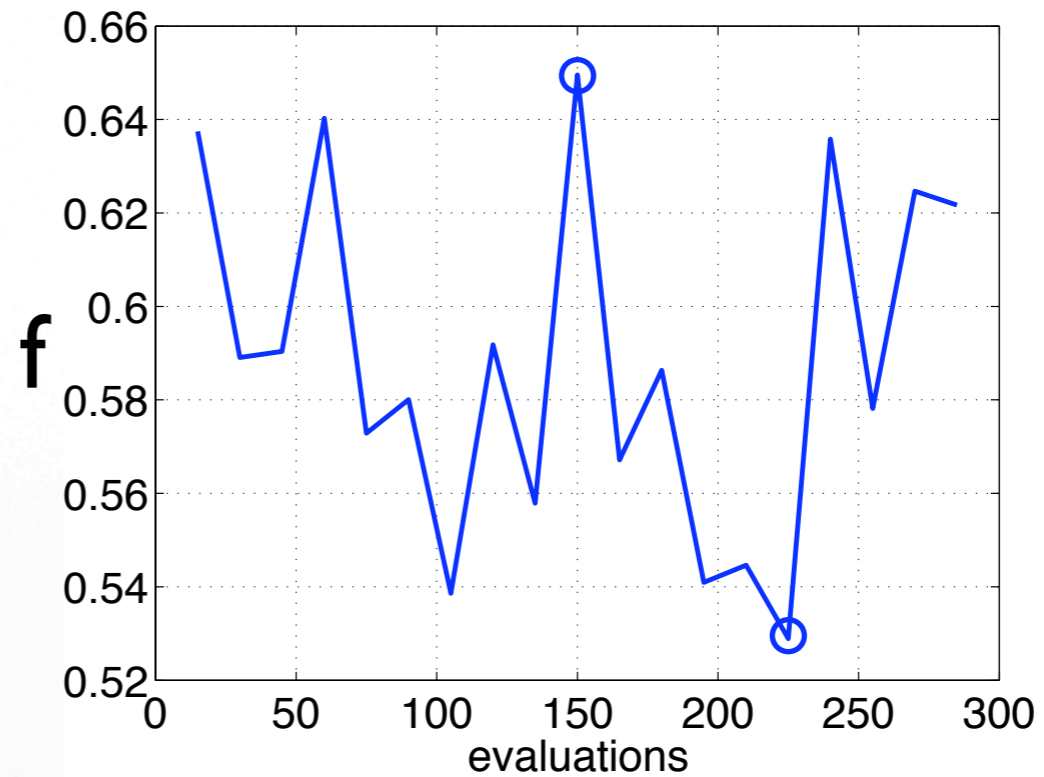
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$\alpha_1$   
 $\lambda$   
 $\arctan(\epsilon_2, \epsilon_1)$   
 $\delta$   
 $\sqrt{\epsilon_1^2 + \epsilon_2^2}$   
 $\Gamma_2/\Gamma_1$   
 $\alpha_2$

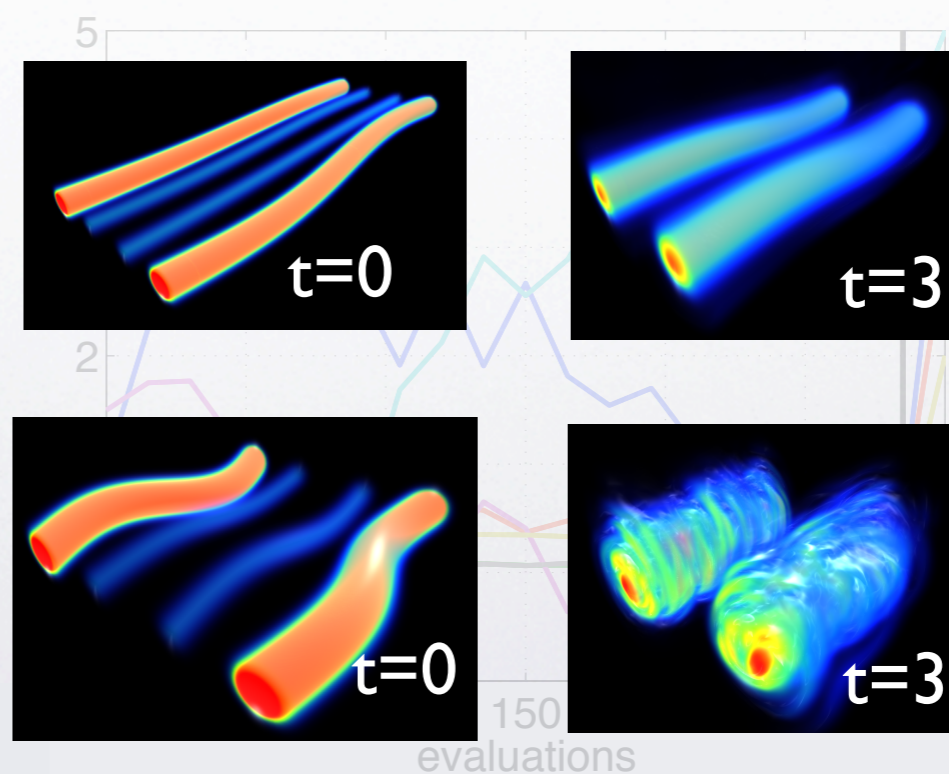
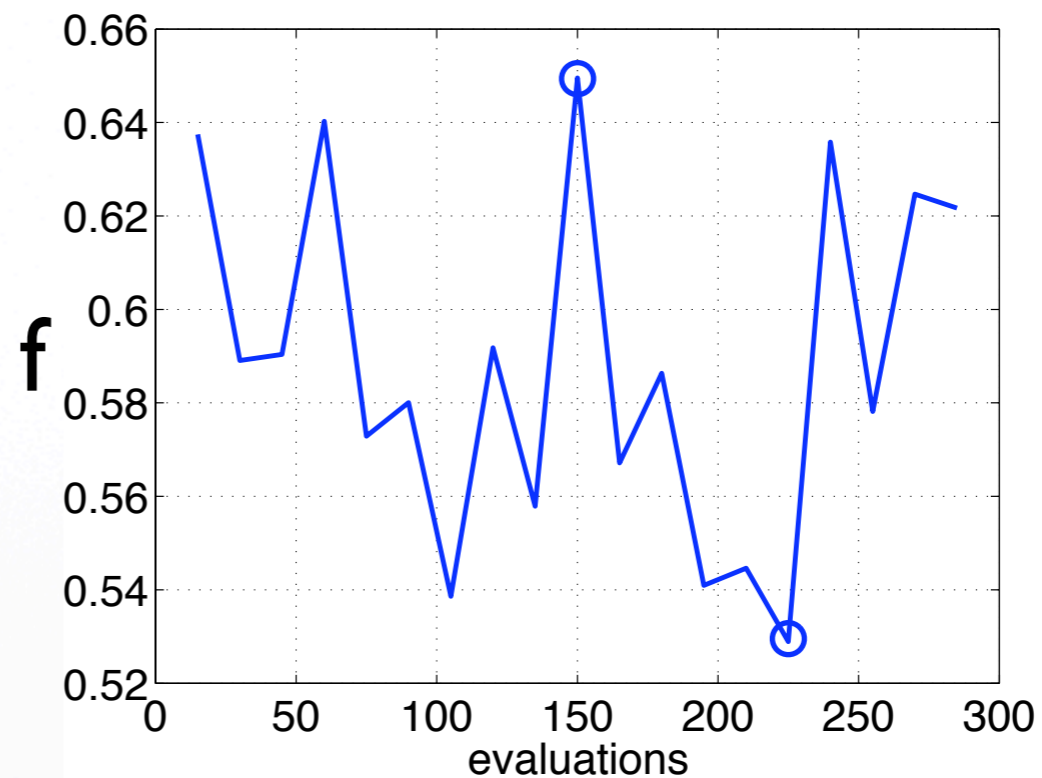
# Optimization of Vortex Decay

- PRESENT

- After sweep toward large  $\lambda$ , moves toward smaller values
- Convergence

- PROBLEMS

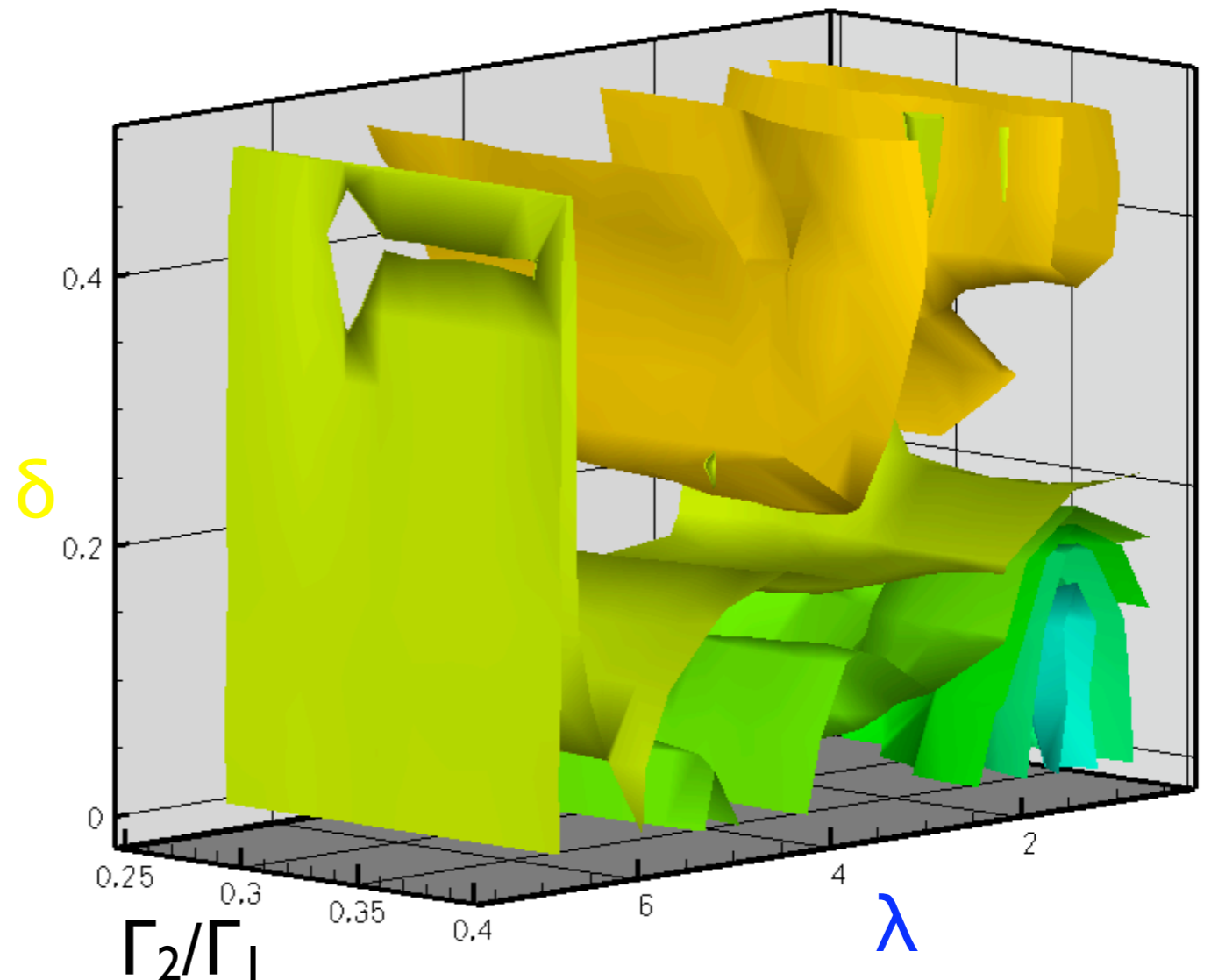
- Resolution of Optimized Flows ?
- Synchronize Optimization and Numerics





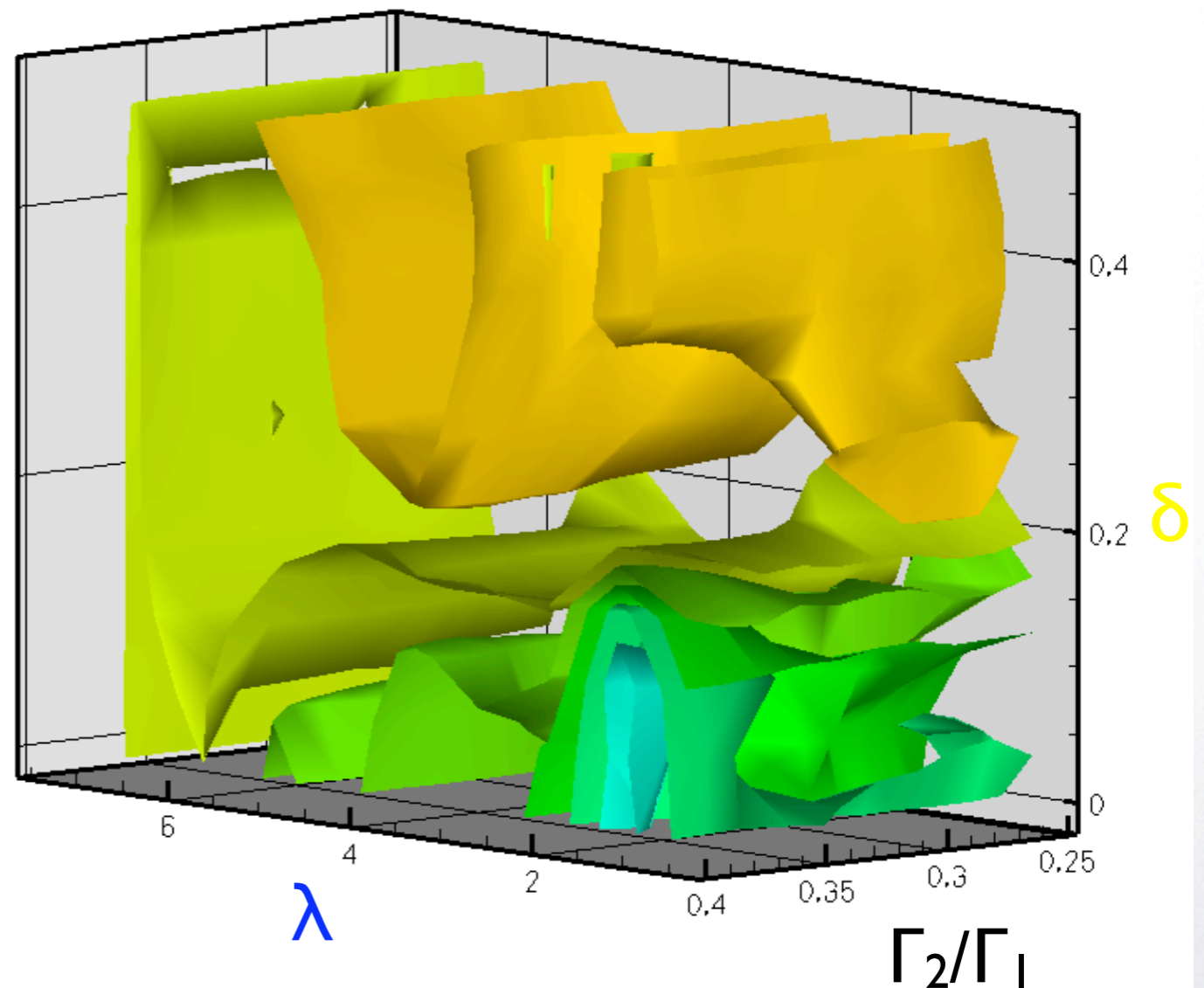
# Optimization of Vortex Decay

- FUNCTION LANDSCAPE
- in  $(\lambda, \delta, \Gamma)$  hyperplane
- Multi-modality
- Effect of numerics?



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- FUNCTION LANDSCAPE
- in  $(\lambda, \delta, \Gamma)$  hyperplane
- Multi-modality
- Effect of numerics?



# Ongoing work

- Development
- *Bigger, faster:*  
Mixed MPI/SMP implementation of library
  - multicore machines: BlueGene/P
- *Smarter:*  
Unbounded boundary conditions
- *More physical:*  
Non-periodic streamwise boundary conditions
  - spatially developing wake

# Conclusions

- Implementation of Vortex Particle Method on massively parallel architecture
  - Scalability
- Large-scale High Re DNS
- Coupling of DNS code with Evolutionary Strategy: Optimization of wake decay

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Swiss Super-Computing Center