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**Resampling**

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- At the last step, we obtain samples from the target distribution.

$$p_k(\vartheta \mid d) \propto p(d \mid \vartheta)^{\gamma_k} \pi(\vartheta)$$

**Sampling Importance Resampling**

**Annealing**

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## Sampling Importance Resampling

1. If I have samples for $\gamma_1 = 0$ then using **SIR** I have samples approximately for $\gamma_2$

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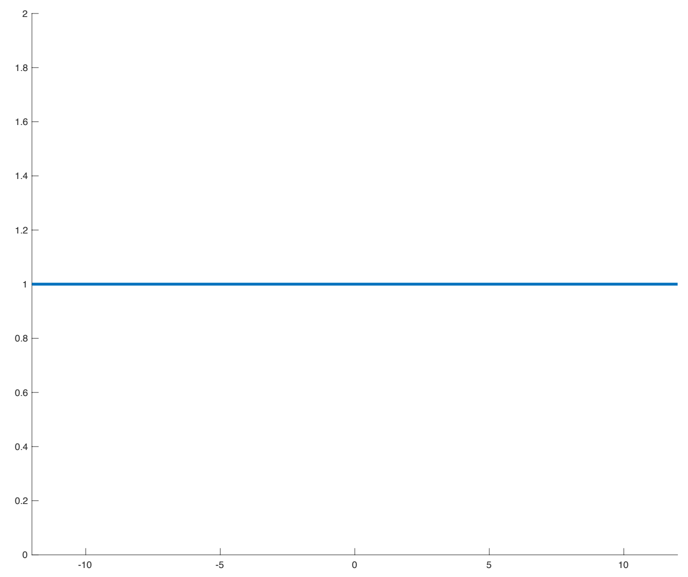

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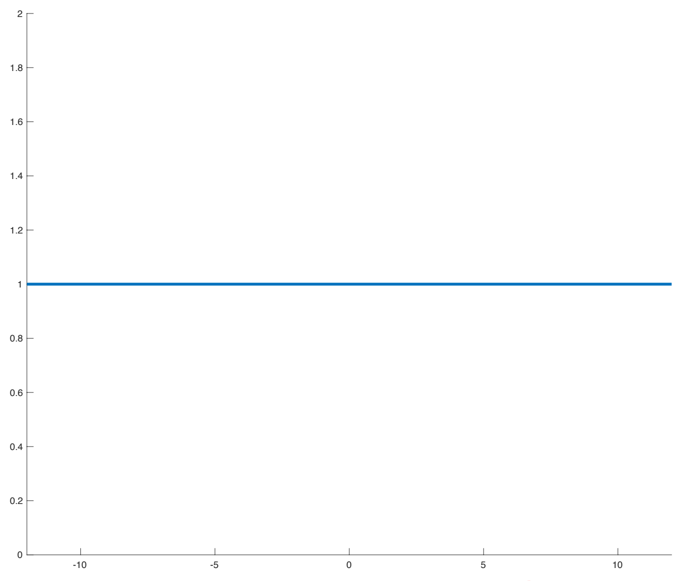

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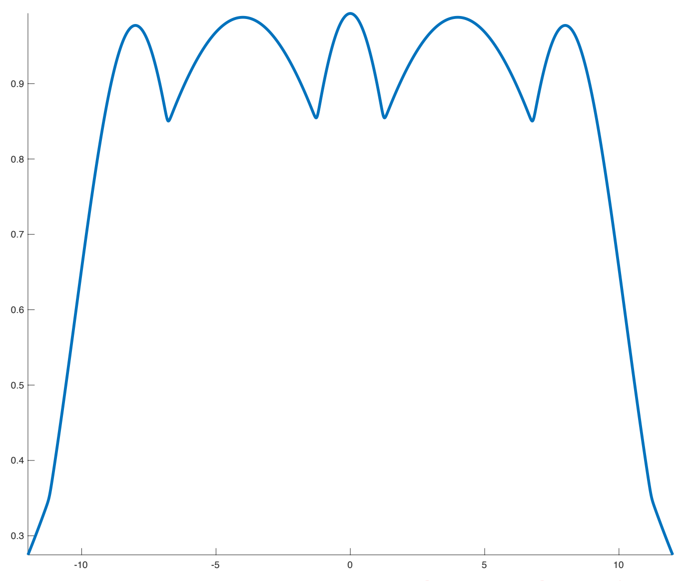

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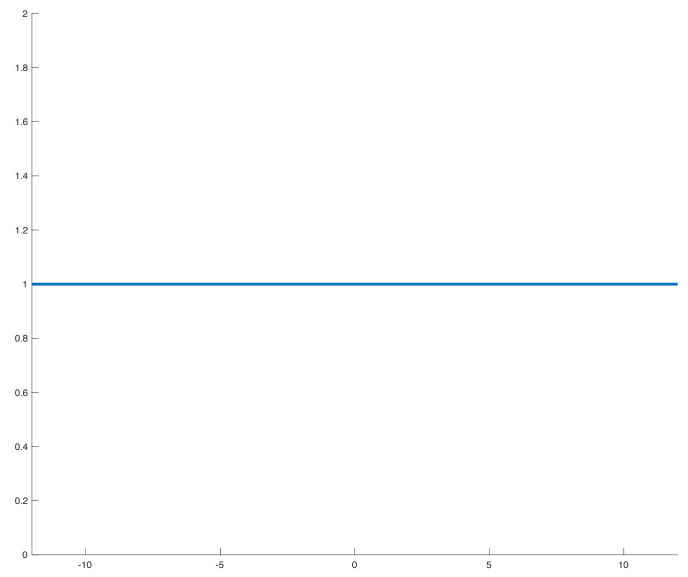

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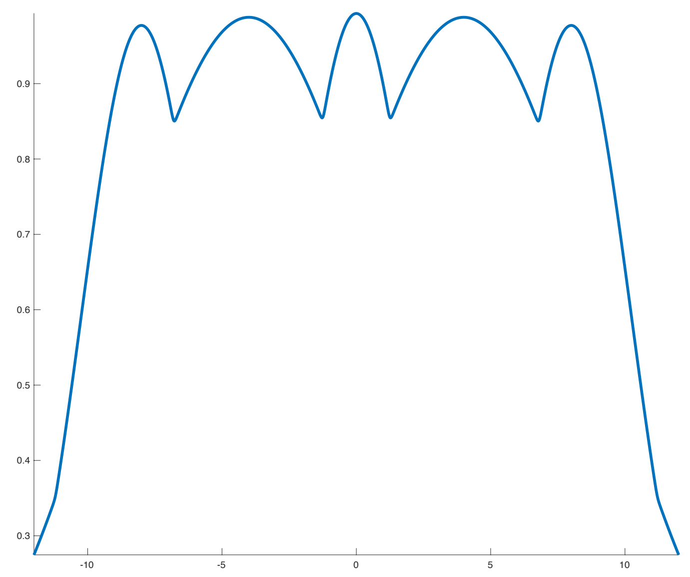

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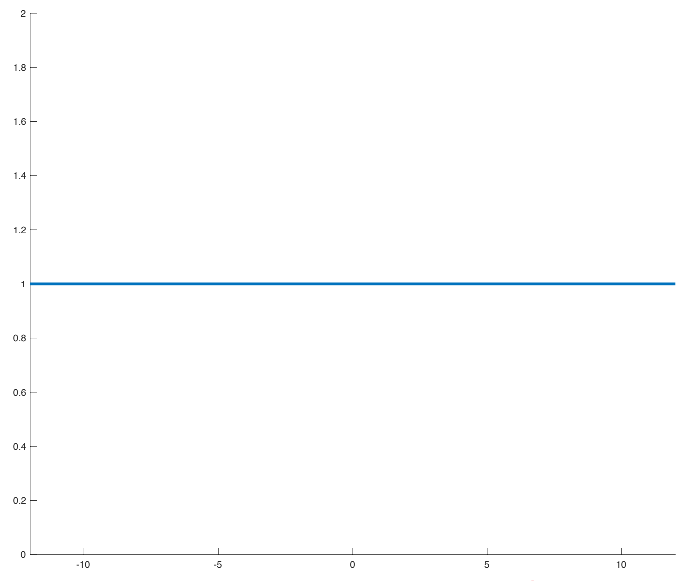

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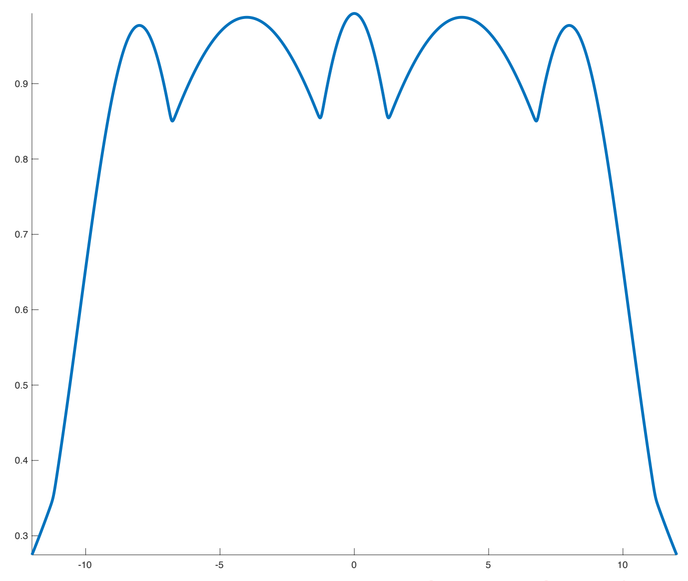

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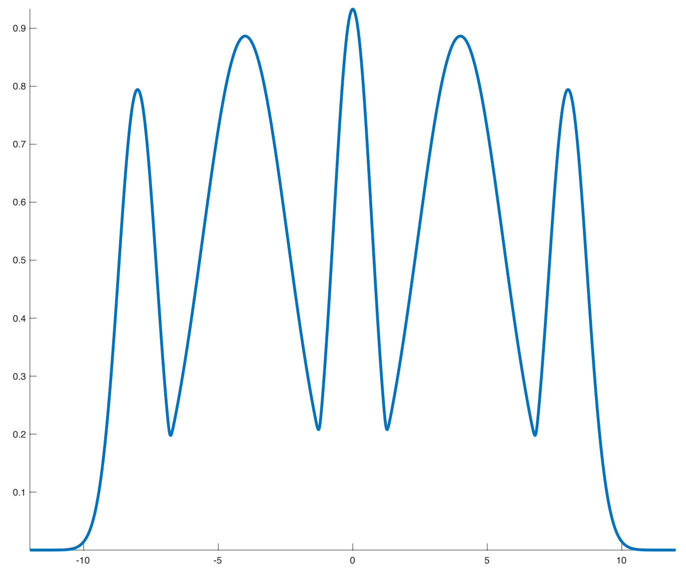

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N. If I have samples for $\gamma_{N-1}$ then using **SIR**
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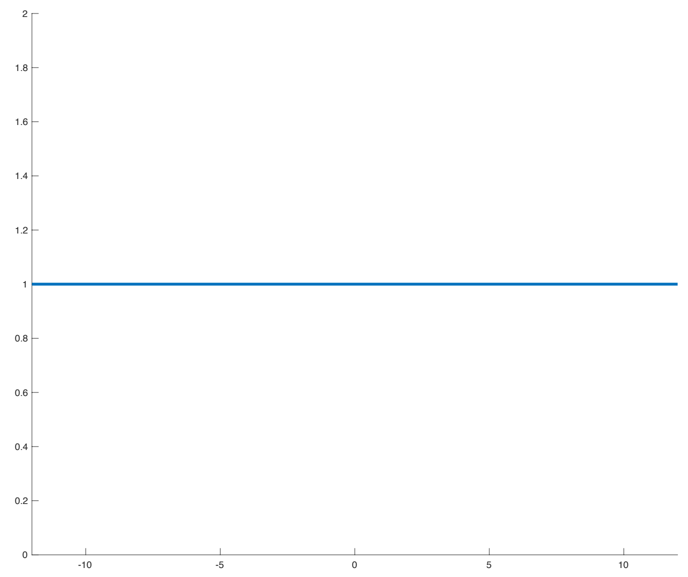

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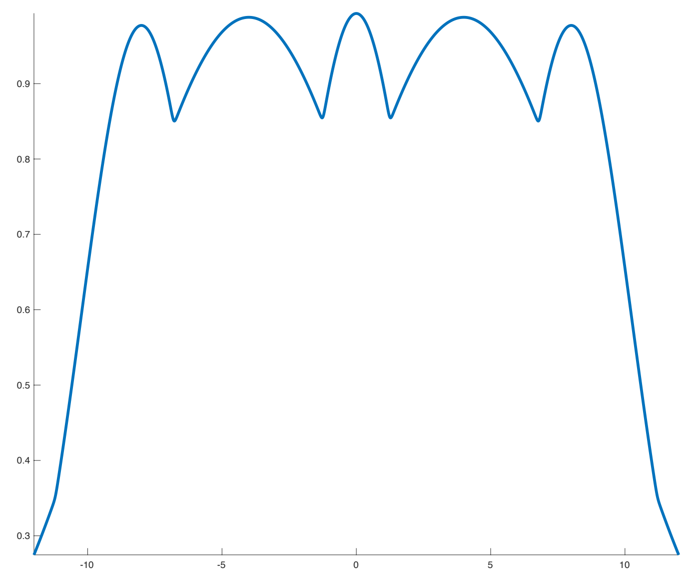

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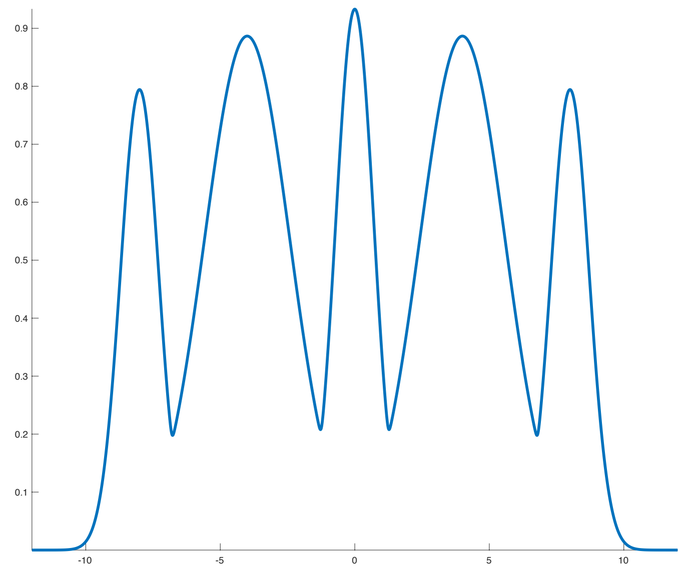

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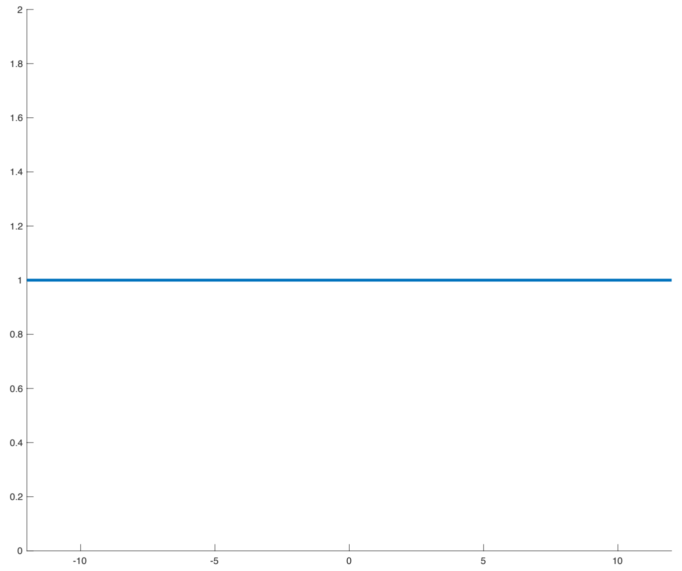
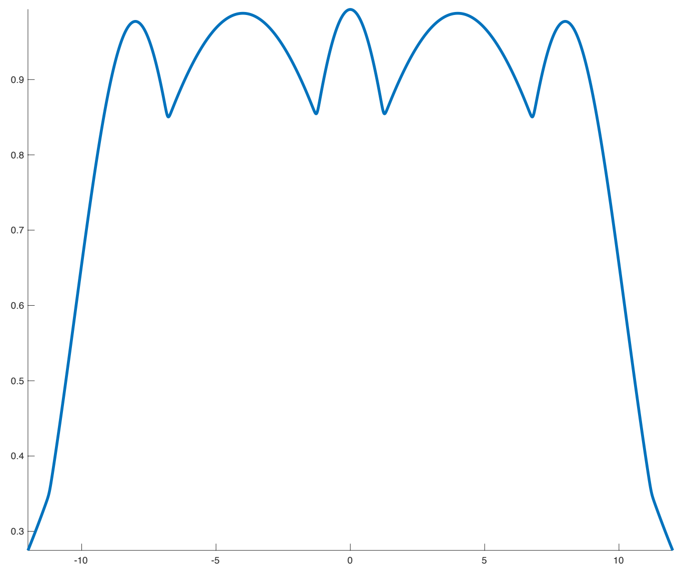


$\gamma_1 = 0$

→



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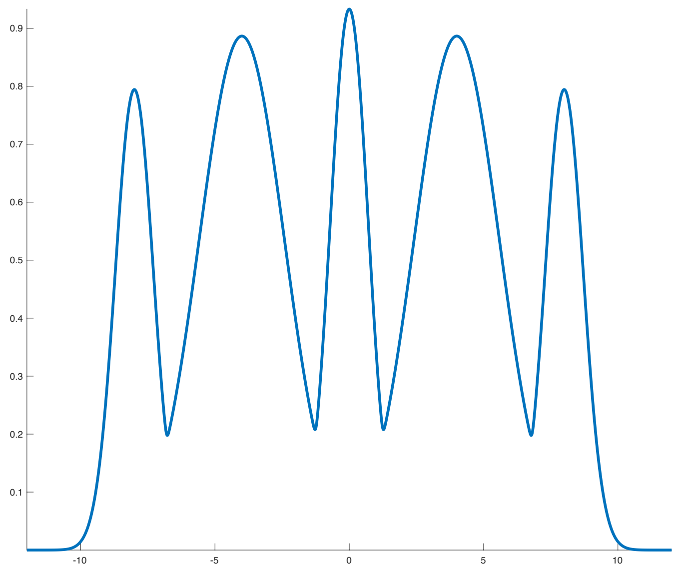
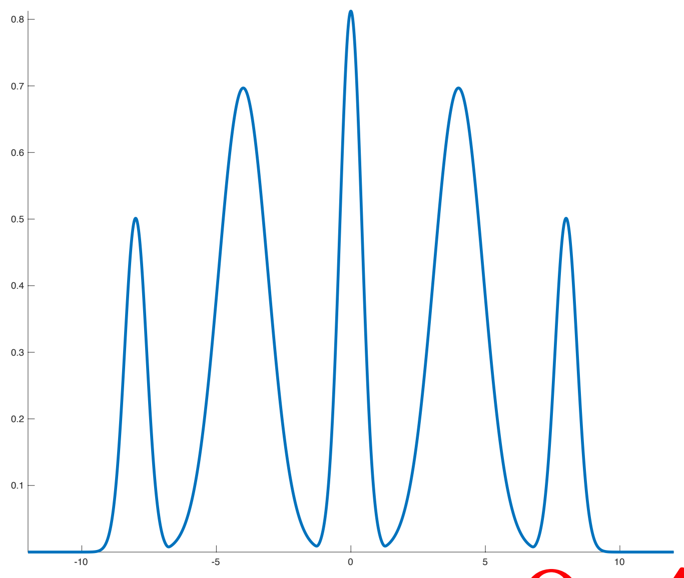


$\gamma_3 = 0.1$

→ . . .

→



$\gamma_{N-1} = 0.4$

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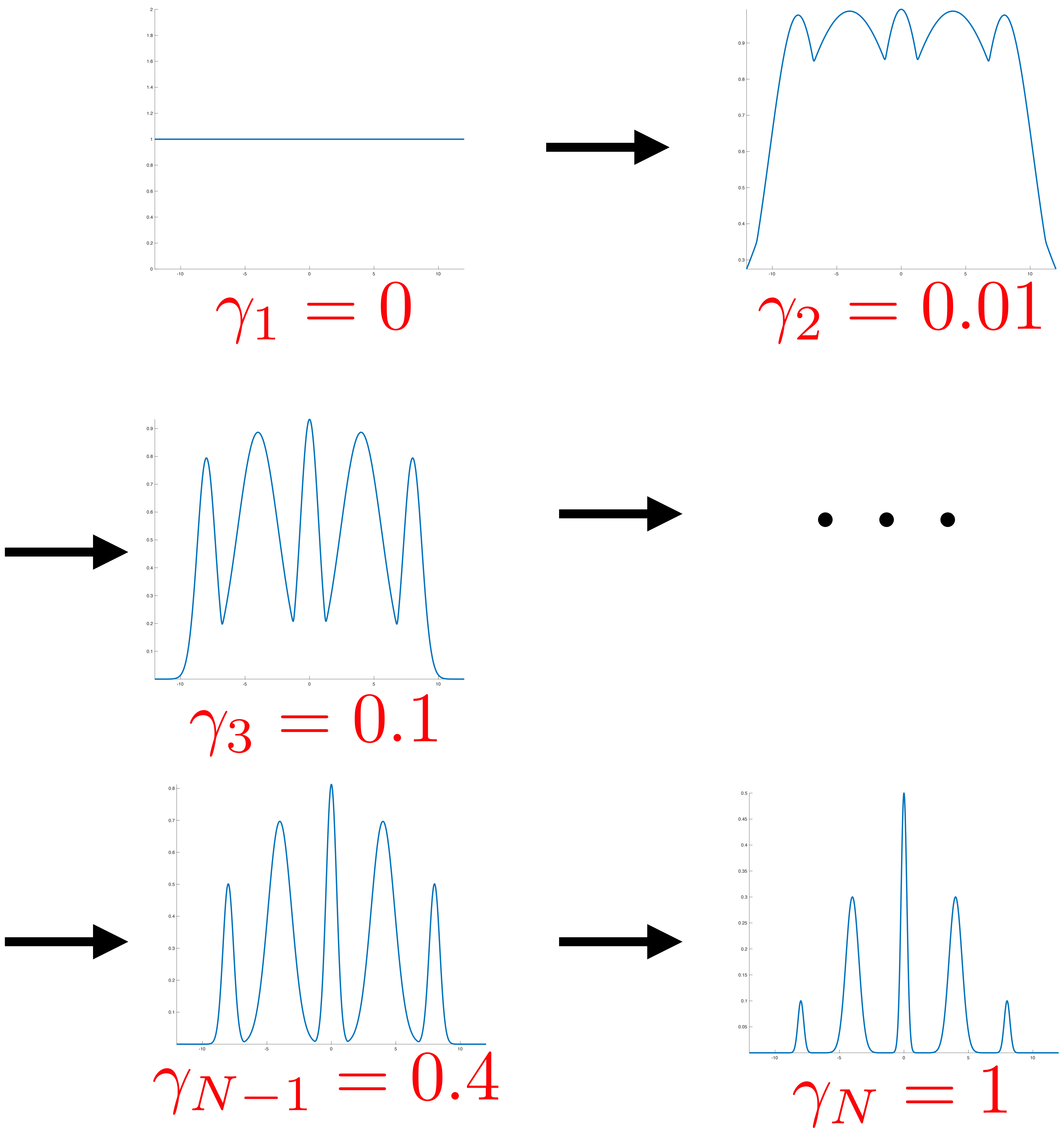

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